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XVI. *Time Measurements of Magnetic Disturbances and their Interpretation.* *By* C. Chree, *Sc.D., LL.D., F.R.S.*

Received February 19, 1914.

§ 1. Fresh interest in the question of the simultaneity or practical simultaneity of occurrence of magnetic disturbances over the whole earth was aroused a few years ago by the advancement of a theory, due to Dr. L. A. Bauer, expressed by himself in the following words*: "Magnetic storms do not begin at precisely the same instant all over the earth. The abruptly beginning ones, in which the effects are in general small, appear to progress over the earth more often eastwardly, though also at times westwardly, as a speed of about 100 km. to 200 km. per second, so that if a complete circuit of the earth were made it would require, on the average, between seven and three minutes."

A discussion by Mr. R. L. Faris† of the apparent times of commencement of 15 magnetic storms at the five observatories of the U.S. Coast and Geodetic Survey appeared favourable to Dr. Bauer's views, giving for the mean time of propagation round the earth about 3½ minutes.

In a Paper‡ read before this Society in November, 1910, I discussed Mr. Faris's Paper critically, reaching the conclusion that his results were insufficient to confirm Dr. Bauer's theory, which appeared to me on various grounds to be somewhat improbable.

At the end of my Paper, *l.c.*, p. 56, I said: "I think Dr. Bauer's conclusions could be adequately tested by a careful comparison of curves from selected stations fairly encircling the globe, choosing, if possible, stations whose time measurements are specially reliable."

Shortly afterwards, on December 20, 1910, Dr. Bauer§ issued a circular to observatories requesting recipients to send to the Carnegie Institution of Washington "the Greenwich mean civil times for your observatory of sudden beginnings of the 15 magnetic disturbances tabulated by Mr. R. L. Faris in 'Terr. Mag.,' Vol. XV., p. 101." Dr. Bauer also asked for particulars of the amplitude of the disturbances in the several magnetic elements, a side of the question which I do not at present propose to touch on.

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* "Terrestrial Magnetism," Vol. XV., 1910, p. 232.

† "Terrestrial Magnetism," Vol. XV., p. 93.

‡ Physical Society "Proceedings," Vol. XXIII., p. 49.

§ "Terrestrial Magnetism," Vol. XVI., p. 85.

A large number of observatories responded to Dr. Bauer's appeal, and the data sent him appeared in the June and September, 1911, issues of "Terrestrial Magnetism." Some observatories—*e.g.*, Greenwich—measured both the times and amplitudes themselves; others, including Potsdam, measured the times, but sent copies of the curves to Washington, so that the amplitudes might be measured there. For my part, I sent rough tracings of the Kew D (declination) and H (horizontal force) curves, indicating what points had been taken as representing the commencements of the disturbances, because in a good many cases there seemed a possibility of confusion.

The September, 1911, issue of "Terrestrial Magnetism" contained a new set of time data for the five U.S. Coast and Geodetic stations, preceded by the following explanation from Dr. Bauer (*l.c.*, p. 200): "On March 16, 1911, the following self-explanatory request was made . . . to the superintendent of the U.S. Coast and Geodetic Survey: 'In addition to the data . . . already supplied, kindly furnish time scalings of all the disturbances, . . . taking into consideration the specimen curves from Potsdam and Kew, the tracings of which are enclosed herewith. Since the Kew curves are subject to a possible electric car disturbance, I would suggest that chief consideration be paid to the Potsdam curves, the precise point of scaling of which has been indicated by the observer (Dr. Venske).' "

Of the new data thus supplied some differ considerably from those supplied originally. The principal reason for calling attention to this matter here is because it is important to know that the final time data for the five U.S. stations were got out under the immediate direction of Mr. Faris, to whom the Potsdam curves had been supplied for guidance. This would naturally go a long way to secure that the disturbances whose times were measured really corresponded. Also, as Mr. Faris, it will be remembered, was originally on the whole distinctly in favour of Dr. Bauer's theory, the fact cannot be regarded as likely to prejudice a decision favourable to that theory.

§ 2. The June, 1911, issue of "Terrestrial Magnetism" held out hopes that "the final compilation and discussion" would follow at an early date, and though a considerable time has since elapsed I should not have intervened even now but for the appearance of a Paper by Dr. G. Angenheister,* which discusses all the time data published by Dr. Bauer, along with some additional data obtained from quick runs of magnetographs taken at four stations in September, 1911.

Dr. Angenheister, following a similar procedure to that originally adopted by Mr. Faris, arranged the 33 stations which responded to Dr. Bauer's invitation in four groups, according to their longitude. The mean longitudes of the groups were respectively 76°, 175°, 274° and 354° west of Greenwich. Confining himself entirely to results from the H curves, Angenheister calculated the mean times of apparent commencement of each disturbance for the four groups of stations separately, and concluded that any differences in these times lay inside the limits of error to be expected with magnetographs run at the ordinary slow rate.

Dr. Angenheister's discussion will, I think, convince most people that the theory of a time of travel round the earth, from east to west or west to east, of the order of three minutes is not tenable for any of the 15 disturbances; but there remain a number of interesting points not discussed by him which I propose to consider here.

There is the question how far the time data from D and V (vertical force), which Angenheister did not utilise, agree with those from H. There is also the possibility that disturbances might travel, or appear to travel, from north to south or south to north. Magnetic disturbances are larger and more numerous in high than in low latitudes. Also, in some cases at least, they seem clearly associated with aurora, which is mainly a phenomenon of the Arctic and Antarctic regions. Supposing aurora to represent, as is generally believed, an electric discharge in the upper atmosphere, a gradual increase in the intensity of that discharge might produce a magnetic disturbance which would seem to propagate itself at a finite rate from the auroral belt towards lower latitudes.

Lastly and principally, the data seemed likely to throw a valuable light on the question of the accuracy of time measurements of disturbances as recorded by ordinary magnetographs. The 15 disturbances dealt with were all selected *after* the event, so that the records were taken under absolutely normal conditions. On any given day, of course, an individual station may be exceptionally fortunate or exceptionally unfortunate in the accuracy of the time marks on the curves, and, even in the case of as many as 15 disturbances, a more than usual amount of good or bad fortune may have been experienced at particular observatories; but deductions based on the records of 15 disturbances at some 30 stations cannot but represent a close approach to average conditions.

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* "Nachrichten der K. Gesell. der Wissenshaften zu Göttingen Math Phys. Klasse," 1913.

§ 3. Table I. deals with the problem treated by Angenheister, but in a somewhat different way, making use of D results instead of H. It arranges the observatories, as Angenheister did, in the order of their longitude, but does not divide them into groups, and gives for each, not the absolute time recorded for the commencement of the disturbance, but the difference between that time and the arithmetic mean of the times derived from all the observatories, with one or two exceptions. The exceptions include all the data from Eskdalemuir, Batavia and Mauritius. The two first of these stations were omitted, because they sent data for only two or three of the disturbances (Eskdalemuir was not in existence during the earlier disturbances), and as it was desired to intercompare the results for different disturbances this seemed the fairest course. Mauritius was omitted because a number of the Mauritius times seemed so abnormal as to suggest errors of the order of 10 minutes. If the Mauritius times are really correct, they represent something wholly abnormal. Two or three individual data from other stations were omitted for one reason or another, but the blanks in Table I. represent in all but a few cases simply the absence of D data. Toungoo, Barrackpore and Kodaikanal in particular contributed few D data, giving their principal attention to H.

The storms are numbered 1 to 15 in chronological order. The dates are given at the top of the table, but are omitted in the subsequent tables. The times given represent the arithmetic means of the accepted data from all the stations combined, and the figures in the columns below represent the differences, in minutes, between the times assigned at the individual stations and these arithmetic means. A plus sign indicates that the corresponding reading was later than the mean.

If the storms travelled round the earth at such a rate as Dr. Bauer supposed, we should have plus entries grouped together, tending towards a maximum at some one meridian, and negative values grouped together tending towards a numerical maximum at an opposite meridian. There seems no trace of this

Table I.—*Times of Commencement from Declination Curves.*

Disturbance No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Date		July 29, 1906.	Aug. 7, 1906.	Dec. 21, 1906.	Feb. 9, 1907.	July 10, 1907.	Oct. 13, 1907.	March26, 1908.	Aug. 19, 1908.	Sept.11, 1908.	Sept.11, 1908.	Sept.28, 1908.	Sept.29, 1908.	May 14, 1909.	Sept.25, 1909.	Sept.25, 1909.
Hour		H. M. 19 56·6	H. M. 13 38·9	H. M. 21 31·0	H. M. 14 12·9	H. M. 14 22·9	H. M. 7 42·9	H. M. 17 42·2	H. M. 0 14·5	H. M. 7 20·2	H. M. 21 46·8	H. M. 8 41·6	H. M. 1 32·0	H. M. 4 56·4	H. M. 8 38·8	H. M. 11 40·9
Station.	Long.W.															
Greenwich	0° 0′	+0·4	−3·9	0·0	+1·1	+0·1	+1·1	+0·8	+0·5	−0·2	+0·2	+0·4	+1·0	+1·6	+3·2	−0·9
Kew	0° 19′	−0·1	+0·1	+2·0	−0·9	−0·9	+0·1	+0·8	+0·5	−1·2	+0·2	−0·6	−3·0	−2·4	−0·8	+1·1
Stonyhurst	2° 28′	−0·8	−1·4	...	+2·7	−0·4	+0·2	...	−2·3	−2·8	−2·0	0·0	−0·9	...	...	...
Falmouth	5° 5′	+1·4	+0·1	0·0	+0·1	−0·9	−0·9	+0·8	+0·5	+0·8	+1·2	+1·4	−3·0	−0·9	−0·8	...
Pilar	63° 53′	+1·7	−0·9	−1·2	−1·6	...	+1·7	−0·3	−1·0	+0·4	−1·6	−0·2	+0·9	+0·5	−0·4	+0·4
Porto Rico	65° 26′	+0·1	...	+2·9	+0·9	−0·2	−0·3	−0·5	+0·4	+0·4	+0·8	+0·1	+0·7	−0·9	+3·0	+0·3
Cheltenham	76° 50′	−0·5	−0·1	+0·6	+0·2	+0·3	−0·9	+1·6	+0·3	−0·2	+0·4	−0·6	−0·7	+0·8	+1·4	+0·1
Agincourt	79° 16′	−1·0	+1·0	0·0	+0·1	+1·8	0·0	−0·1	−0·1	+0·5	+0·9	+0·2	−1·0	+1·4	−1·5	−1·2
Baldwin	95° 10′	−1·0	+0·3	+0·4	+0·1	+0·3	−0·1	−0·1	−0·1	+0·8	−0·8	−0·5	−0·2	+0·8	−0·2	+0·5
Sitka	135° 20′	−0·3	+1·8	−0·8	+1·1	−0·7	+0·3	...	+0·7	+0·8	−0·4	+1·4	...	+0·4	+1·4	+0·6
Honolulu	158° 4′	−2·0	−1·3	−0·4	0·0	+0·7	−0·3	−1·8	+0·6	+1·6	0·0	+2·2	−0·2	−1·3	+0·2	+0·7
Samoa	171° 45′	+0·5	+0·7	−3·5	−0·1	−1·2	...	−0·2	−0·5	...	...	...	+1·9	...	...	...
Zi-ka-wei	238° 34′	−2·6	+2·1	...	−1·9	−0·9	+3·1	...	...	−1·2	−0·8	...	−1·0	+0·6	−0·8	−2·9
Buitenzorg	253° 11′	−0·5	−2·6	−0·4	−0·9	−0·1	−1·2	−3·2	+0·5	−2·5	−0·9	+0·4	+1·6	0·0	+0·5	+1·1
Toungoo	263° 33′	+1·3	−1·0	+0·4	...	−0·1	+0·7	...	...	...	...	...	...	...	...	...
Barrackpore	271° 38′	...	...	+0·2	...	+1·2	...	...	−0·2	...	0·0	...	+1·2	...	...	−0·8
Dehra	281° 57′	+1·1	−0·9	+1·1	+2·1	−0·7	−0·9	−0·8	−0·2	+1·1	−0·5	...	+1·1	...	...	+0·8
Kodaikanal	282° 32′	...	...	...	...	...	−0·4	...	...	...	...	...	...	...	...	−0·1
Bombay	287° 8′	−0·2	−0·5	−1·8	−0·5	−0·7	+0·3	−1·5	−0·8	+0·5	−0·4	−0·7	−1·1	−2·0	−1·1	+0·5
Ekaterinburg	299° 22′	+1·4	+2·3	+0·7	...	+0·4	+1·1	+1·8	−0·3	...	...	−0·7	+1·0	+1·1	−0·8	−0·2
Helwan	328° 39′	...	...	...	...	+1·1	+2·1	...	+0·5	+0·8	+1·2	...	0·0	+0·6	+0·2	+1·1
Pola	346° 9′	...	+0·5	...	+0·3	+2·6	−0·1	+2·1	+3·0	...	...	...	+2·1	+1·5	+1·2	+1·5
Potsdam	346° 56′	+0·9	+0·7	0·0	−0·5	+0·4	−0·8	+0·2	−0·5	−0·1	−0·4	−1·1	+0·2	−0·4	−0·5	+0·1
Rude Skov	347° 33′	...	...	...	...	−0·4	−1·4	+0·6	...	−0·1	+0·3	−1·1	−0·6	−1·0	−1·2	...
Munich	348° 23′	+0·5	+0·6	−0·8	−0·6	+0·3	+1 1	−0·2	+0·7	−0·2	−0·3	−0·5	+0·2	−0·7	+0·1	−2·0
Wilhelmshaven	351° 51′	−1·2	0·0	...	−2·5	−2·5	−3·0	+0·2	−2·1	+0·2	+0·6	−1·2	−2·1	0·0	−1·9	−1·5
De Bilt	354° 49′	...	−0·3	...	...	...	+0·6	...	−0·5	+1·6	+1·3	−0·6	−0·5	−1·1	−0·8	+0·6
Uccle	355° 39′	...	+1·8	+0·8	+0·6	−0·1	+0·5	...	−1·0	+0·7	+1·6	+0·6	+0·1	−0·3	+0·9	+2·5
Val Joyeux	357° 59′	...	+0·3	−1·0	−1·5	+0·1	−3·1	...	+2·2	−0·2	+0·3	+0·1	+1·5	+1·4	+0·5	+0·2
Tortosa	359° 30′	...	...	+0·4	+2·3	+0·2	−0·1	+0·2	−1·2	+0·2	+0·2	...	+1·6	...	−2·1	−1·7

Table II.—*Times of Commencement from Horizontal Force Curves.*

Disturbance No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hour		H. M. 19 56·5	H. M. 13 38·0	H. M. 21 30·6	H. M. 14 13·2	H. M. 14 22·8	H. M. 7 43·1	H. M. 17 41·5	H. M. 0 14·4	H. M. 7 20·5	H. M. 21 46·7	H. M. 8 41·6	H. M 1 32·0	H. M. 4 55·2	H. M. 8 38·2	H. M. 11 40·7
Station.	Lat. N.															
Sitka	57° 3′	−0·5	+2·2	0·0	+0·3	−0·4	+0·9	...	+1·3	+0·5	+0·1	+1·3	...	+1·9	+1·6	+0·3
Ekaterinburg	56° 49′	+0·2	+0·8	+0·9	+0·8	+0·5	+2·5	+2·0	+0·1	...	...	+1·2	−0·2	+1·4	+0·6	−0·7
Rude Skov	55° 51′	...	...	...	...	+0·7	...	+0·4	−1·1	+0·1	+0·8	+1·8	+1·1	...	−0·5	...
Stonyhurst	53° 51′	−0·8	−0·5	−2·5	+1·9	−0·3	−0·4	...	−2·8	−2·7	−1·5	−0·4	+2·0	−0·9	...	...
Wilhelmshaven	53° 32′	−0·1	+1·9	+0·8	−2·8	−0·9	−1·7	−1·1	+1·0	−0·6	−0·3	−0·2	−0·6	...	+0·2	...
Potsdam	52° 23′	0·0	−0·5	−0·6	−0·8	+0·4	+0·6	−0·2	+0·1	+0·2	+0·9	+1·0	0·0	+0·6	−0·4	+0·3
De Bilt	52° 5′	−0·1	−0·4	...	...	...	−0·6	−0·5	−0·4	+0·7	−0·6	−0·6	−0·5	+0·1	−0·2	+0·3
Greenwich	51° 28′	+0·5	−4·0	−0·6	+0·8	+0·2	+0·9	+1·5	+0·6	−0·5	+0·3	+0·4	+1·0	+2·8	+3·8	−0·7
Kew	51° 28′	+0·5	+1·0	+0·4	−0·2	−0·8	−0·1	+1·5	+0·6	−2·5	+0·3	−0·6	−3·0	+1·8	−0·2	+0·3
Uccle	50° 48′	+0·5	+1·5	+2·3	−1·6	−0·4	+3·7	−0·8	−0·1	+0·8	−1·8	+1·0	+0·1	−0·6	+0·4	+0·7
Falmouth	50° 9′	+0·5	0·0	+3·4	+1·8	+1·2	+1·9	+2·5	+0·6	+1·0	−1·2	+0·4	−1·0	−0·2	−2·2	−1·7
Val Joyeux	48° 49′	−0·5	−1·1	−0·6	−1·8	+0·2	−4·5	−3·5	+1·7	−0·5	−0·8	−0·5	+1·5	+1·4	+1·1	+0·4
Munich	48° 9′	−0·2	+1·5	0·0	−1·2	−2·2	+1·9	−1·0	+0·5	+2·0	−0·7	+1·2	+0·4	...	+0·3	−1·7
Pola	44° 52′	+1·1	−2·0	−2·0	−1·3	+2·3	−2·8	−1·7	−0·8	...	...	−0·5	−0·9	−2·9	−1·1	−1·3
Agincourt	43° 47′	−0·2	+2·3	+0·4	−0·7	−0·3	+0·1	+1·3	−0·2	+0·6	+1·0	+0·2	−1·7	+2·1	+0·5	+0·8
Tortosa	40° 49′	...	...	...	+3·7	−0·3	−2·7	+0·1	−2·3	−0·8	−1·2	...	+1·0	...	−1·1	+0·5
Baldwin	38° 47′	+0·6	+1·2	0·0	−0·2	+0·4	+0·2	+0·7	+0·5	+0·3	+2·4	0·0	−0·4	−0·5	−0·5	−0·2
Cheltenham	38° 44′	0·0	+1·0	+1·2	0·0	−0·1	−0·1	+3·1	0·0	−0·4	0·0	−0·3	−0·8	+1·2	+0·8	+0·5
Zi-ka-wei	31° 12′	−2·5	−1·0	−0·6	−2·2	−0·8	+0·9	...	+0·6	−1·5	−1·7	−2·6	+1·0	−1·2	−1·2	−0·7
Dehra	30° 19′	+0·4	−0·1	+0·9	+1·0	−0·4	+0·5	−0·6	0·0	+2·3	+0·4	+0·6	−0·1	...	−1·2	−0·2
Helwan	29° 52′	...	...	...	...	−1·8	+0·9	+0·5	+1·6	+1·5	+2·3	−2·6	+1·0	−1·2	−0·2	−1·7
Barrackpore	22° 46′	+0·5	−0·3	+0·7	...	+0·6	−0·2	0·0	−0·3	...	+1·0	−0·3	+1·9	...	−0·4	−0·5
Honolulu	21° 19′	−2·2	−0·8	−2·3	−1·2	−1·9	−1·0	−1·3	−0·6	+1·1	−0·7	+1·5	0·0	−0·4	−0·3	−0·1
Toungoo	18° 56′	+0·8	−0·2	−0·5	+0·5	+1·7	−0·2	−0·5	...	...	...	...	...	...	−0·8	−0·5
Bombay	18° 39′	−0·1	+0·2	−0·4	−0·1	−0·4	+0·8	−0·1	−1·2	+0·2	−1·3	−0·2	−0·8	−1·5	−0·2	+1·5
Porto Rico	18° 9′	−0·1	−1·3	+3·4	−0·1	−0·2	+0·3	−0·1	+0·8	+0·0	+0·5	−0·8	−0·2	−1·0	+0·4	+0·1
Kodaikanal	10° 14′N.	+1·3	−0·1	+0·5	+1·6	+1·9	0·0	0·0	0·0	+2·7	+1·9	+0·6	+0·5	...	−0·7	+1·2
Buitenzorg	6° 35′S.	−0·4	−1·7	−0·6	−1·2	+0·3	−2·3	−3·1	+0·3	−1·6	+0·4	+1·0	+2·8	0·0	+0·5	+1·9
Samoa	13° 48′	+0·4	−0·4	−3·1	−0·4	...	+0·7	+0·5	−0·1	+0·1	+1·0	−0·3	−0·4	...	...	...
Pilar	31° 40′	...	...	−0·5	+2·3	+0·8	−0·5	+1·0	−1·4	−3·4	−2·3	−2·6	−3·3	−2·8	+0·2	+0·6

Table III.—*Times of Commencement from Vertical Force Curves.*

Disturbance No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hour	H. M. 19 56·5	H. M. 13 39·2	H. M. 21 31·8	H. M. 14 13·5	H. M. 14 23·6	H. M. 7 43·4	H. M. 17 42·4	H. M. 0 14·7	H. M. 7 22·2	H. M. 21 47·0	H. M. 8 42·6	H. M. 1 32·8	H. M. 4 55·4	H. M. 8 38·4	H. M. 11 41·2
Station.															
Sitka	+0·7	+1·4	−1·6	0·0	0·0	−0·9	...	+1·1	−0·5	0·0	0·0	...	+1·4	+1·6	...
Ekaterinburg	+1·8	+1·1	+1·8	+4·9	+0·9	...	+1·2	−0·2	...	...	+1·1	−0·7	+0·4	+1·5	−0·9
Rude Skov	...	...	...	...	+0·6	...	...	...	...	+0·2	−0·2	−0·4	...	...	...
Stonyhurst	...	...	...	...	−1·1	...	...	−2·0	...	−2·2	−1·0	+1·3	...	...	...
Potsdam	+0·2	+0·5	+0·2	+0·4	+1·0	−0·1	−0·9	+1·1	−1·0	+0·4	+0·4	−0·1	+2·2	+0·1	+0·9
De Bilt	+0·9	...	...	...	...	...	...	...	...	+1·1	−0·6	+1·2	−0·1	...	...
Greenwich	+0·5	−4·2	+1·2	+0·5	−0·6	+0·6	+0·6	+1·3	−1·2	+1·0	−0·6	+1·2	+2·6	...	−1·2
Uccle	...	...	...	−0·7	...	...	...	...	...	−0·5	...	−0·7	+1·5	+1·3	+1·4
Falmouth	+2·5	...	+3·2	+1·0	+4·4	+4·0	+4·6	+2·8	+3·8	−1·0	+4·4	+3·2	−7·4	−3·9	+0·8
Val Joyeux	+0·1	+1·1	+1·9	+0·6	+3·0	−0·6	...	+5·0	+2·6	+1·9	+2·7	+3·1	+6·0	...	+1·1
Pola	...	...	...	−1·5	...	−3·4	...	−0·7	...	...	−1·6	−1·8	−3·4	...	−2·2
Tortosa	...	...	+0·2	+3·1	−0·8	−1·3	−0·5	−1·7	−1·1	−0·3	...	+1·9	...	−1·4	−2·0
Baldwin	−0·7	+0·1	−1·2	−0·1	−0·9	−0·4	−0·1	−0·5	−1·0	+0·6	0·0	−1·0	+1·7	0·0	+1·0
Cheltenham	−0·7	+2·8	+0·1	−1·1	−0·6	+0·2	+1·3	−0·1	−2·2	+0·3	0·0	−0·9	+1·8	−0·6	+0·5
Zi-ka-wei	−2·5	−0·2	...	−4·5	...	+2·6	...	...	...	...	−0·6	−0·8	−1·4	−0·4	−2·2
Dehra	...	−1·9	...	0·0	−0·7	−0·6	−0·4	−0·2	+0·6	+0·4	...	...	...	...	...
Helwan	...	...	...	...	−1·6	+1·6	+1·6	+0·3	+1·8	+2·0	−0·6	...	−0·4	+0·6	...
Barrackpore	...	...	...	...	−0·6	−1·6	...	−2·4	−0·5	+0·2	+0·4	+1·3	...	...	+1·5
Honolulu	−1·8	−0·8	−3·0	−1·1	−0·3	−1·0	−0·7	−0·9	−1·1	−1·2	+0·8	−0·8	−0·3	+0·2	−0·1
Toungoo	...	...	...	...	−1·9	−1·8	...	...	...	...	...	...	...	...	+0·4
Bombay	−0·3	−1·0	−2·9	−0·6	−0·9	−0·2	−1·2	−0·5	−1·0	−0·8	−1·4	−1·1	−1·5	−0·5	0·0
Porto Rico	...	+3·0	+2·6	−0·2	−1·0	−0·2	−1·5	+0·3	−1·2	−0·2	−0·2	+1·0	−1·0	+0·8	−0·4
Kodaikanal	...	...	...	...	+1·3	+0·2	−0·7	...	+0·5	+1·5	+0·3	−0·4	...	...	−0·8
Buitenzorg	−0·4	−2·3	−1·2	−0·6	+0·4	−0·8	−3·7	...	...	−3·2	−0·4	−1·6	+1·6	+0·9	+1·4
Samoa	...	...	...	...	+1·5	+1·5	...	...	+2·1	+0·4	+0·9	−1·3	...	...	...
Pilar	...	...	−1·6	+0·1	−2·4	+2·2	+0·1	−2·2	−1·4	−1·4	−3·6	−2·2	−3·0	+0·3	+0·3

To face page 141.]

in Table I. The incidence of plus and minus signs appears fortuitous. The stations whose data differ most seem as often to be near stations as remote.

§ 4. Table II. is constructed on similar lines to Table I., but it deals with H times and arranges the stations in order of latitude. As in Table I., the numerical variations in the differences from the mean time shown by the returns from the different stations bear no obvious relationship to their geographical position, plus and minus signs occurring promiscuously. So far as H is concerned, there seems no more evidence of motion of disturbances along meridians than there was along parallels of latitude.

The mean times of commencement given in Table II. agree with those found by Angenheister in the case of disturbances Nos. 4, 5, 6, 8, 10 and 15. In the other cases the algebraic excess of the mean values in Table II. over Angenheister's is as follows :—

Disturbance No. ...	1	2	3	7	9	11	12	13	14
Excess in Table II....	−0·1	−0·1	+0·1	+0·3	−0·1	−0·1	−0·2	+0·4	+0·1

In the case of disturbance No. 14, the times given by Angenheister for Stonyhurst (8 h. 29·0 m.) and Eskdalemuir (8 h. 30·0 m.)—the latter at least of which was apparently not employed in calculating his mean—clearly refer to a disturbance anterior to No. 14, which was noted at various other stations, including Greenwich. The times given by Angenheister for No. 7, at Munich, and No. 10, at Sitka, differ by four and six minutes respectively from those given in "Terrestrial Magnetism," and there are some other differences which may be misprints. In the case of Nos. 8, 9, 10 and 12, Angenheister omits the data given in "Terrestrial Magnetism" for Pilar. They are, perhaps, a little outstanding; but the same is true of some other Pilar data which he employs. In the case of this station Angenheister—presumably for some definite reason—has subtracted 0·1 from the times given in "Terrestrial Magnetism." It has, however, seemed best here to follow "Terrestrial Magnetism"; in any case, the difference is immaterial.

Table III. does for V data what Table II. did for H data. The stations follow the same order as in Table II., but Wilhelmshaven, Kew, Munich and Agincourt are omitted as giving no V data. There were a good many blanks in the V data at most stations, and some of the mean values depend on observations from only 13 or 14 stations. In disturbance No. 1 there is a suggestion of motion from south to north, the first seven entries being all plus and the last six all negative. The numerically largest values, however, both positive and negative, occur in intermediate latitudes, and the phenomenon may be quite accidental.

The Falmouth and Val Joyeux estimates for disturbance No. 13 appear abnormal, but were retained as they roughly neutralise one another in the mean.

§ 5. A comparison of two selected pairs of stations will, I think, show at once the inadequacy of Dr. Bauer's theory to explain the facts, and will bring out the true nature of the differences recorded in Tables I. to III.

The first pair of stations selected for this purpose, Potsdam and Baldwin, possess instruments similar, if not identical, in type, and the curves of the former station served as a guide to the identification of the disturbance times at the latter. The second pair of stations, Greenwich and Val Joyeux, possess magnetographs of widely different types, and the times at the two were got out absolutely independently.

The conclusion originally drawn by Mr. Faris from the data given for the 15 disturbances at the five U.S. stations was that the mean time required to travel 360° in longitude was 3·3 minutes from the D curves, 3·5 minutes from the H curves and 3·6 minutes from the V curves. Accepting 3·5 minutes as a mean of these, we should have for the times of travelling by the shortest route

Between Baldwin and Potsdam (108° 14′)	1·05 minutes.
,, Greenwich and Val Joyeux (2° 1′)	0·02 ,,

The times actually observed are given in Table IV.

A plus sign in Table IV. indicates later apparent occurrence at the first-mentioned (most eastern) station of the pair. Supposing the shortest route taken, this would imply, on Dr. Bauer's theory, a movement towards the east. As Val Joyeux and Greenwich are intermediate geographically between Potsdam and Baldwin, on the short arc, any disturbance which took the short route between the more distant pair of stations must necessarily take the short route between the nearer pair. Also, no storm could possibly have a shorter route between Potsdam and Baldwin than between Val Joyeux and Greenwich, unless it originated between the latter two stations, a remote contingency in view of the small difference in longitude between them. That any large proportion of the 15 storms could have originated between Val Joyeux and Greenwich is inconceivable. Thus, the phenomena exhibited in Table IV., when interpreted by Dr. Bauer's theory, would inevitably imply that the part not merely exceeds the whole, but exceeds it by a great deal.

TABLE IV.—*Time Differences between Selected Stations, in Minutes.*

Disturbance. No.	Potsdam, less Baldwin time.				Val Joyeux, less Greenwich time.			
	D.	H.	V.	Algebraic mean.	D.	H.	V.	Algebraic mean.
1	+1·9	−0·6	+0·9	+0·7	...	−1·0	−0·4	...
2	+0·4	−1·7	+0·4	−0·3	+4·2	+2·9	+5·3	+4·1
3	−0·4	−0·6	+1·4	+0·1	−1·0	0·0	+0·7	−0·1
4	−0·6	−0·6	+0·5	−0·2	−2·6	−2·6	+0·1	−1·7
5	+0·1	0·0	+1·9	+0·7	0·0	0·0	+3·6	+1·2
6	−0·7	+0·4	+0·3	0·0	−4·2	−5·4	−1·2	−3·6
7	+0·3	−0·9	−0·8	−0·5	...	−5·0	...	...
8	−0·4	−0·4	+1·6	+0·3	+1·7	+1·1	+3·7	+2·2
9	−0·9	−0·1	0·0	−0·3	0·0	0·0	+3·8	+1·3
10	+0·4	−1·5	−0·2	−0·4	+0·1	−1·1	+0·9	0·0
11	−0·6	+1·0	+0·4	+0·3	−0·3	−0·9	+3·3	+0·7
12	+0·4	+0·4	+0·9	+0·6	+0·5	+0·5	+1·9	+1·0
13	−1·2	+1·1	+0·5	+0·1	−0·2	−1·4	+3·4	+0·6
14	−0·3	+0·1	+0·1	0·0	−2·7	−2·7	...	...
15	−0·4	+0·5	−0·1	0·0	+1·1	+1·1	+2·3	+1·5
Arithmetic means	0·60	0·66	0·67	0·30	1·43	1·71	2·35	1·50

Supposing disturbances equally likely to originate in any longitude, four out of the 15 might have been expected to originate between Potsdam and Baldwin, and thus to have followed the longer route (251° 46′) between these two stations. Taking, however, the view least unfavourable to Dr. Bauer's theory, as applied to that particular pair of stations—viz., that all the disturbances described the shorter route (108°14′)—we see that the times derived from the D, H and V records separately would imply a rate of propagation nearly double that found by Mr. Faris, while the times derived from an algebraic mean of the results from the three elements would imply a rate of propagation 3½ times that found by him. It will also be seen that, so far as Potsdam and Baldwin are concerned, the signs of the time differences derived from D and H did not agree as often as they differed, so that in the majority of instances there was not even agreement as to the direction in which the disturbance travelled. The conclusion seems inevitable that the phenomena are mainly, if not entirely, due to instrumental or observational causes, and the remainder of the Paper assumes this to be the case.

§ 6. The figures, it should be borne in mind, do not represent the highest accuracy attainable with existing magnetographs run at the ordinary rate. If the days for curve measurements had been selected in advance, some at least of the stations would probably have taken precautions not ordinarily observed. On the other hand, the measurements were probably taken with more than the usual care, and represent higher precision than is customary in purely routine work. Before analysing them it is desirable to consider how times are usually determined on magnetic curves. The nature of the photographic record is shown diagrammatically in Fig. 1. The straight line AH—known as the *base* or *time* line—represents the successive positions of a fixed spot of light reflected from a fixed mirror on to photographic paper wound round a drum turned by clockwork. The curved line A′H′ represents light from the same source reflected from a mirror attached to a magnet. In the case of the D or H instrument the drum has its axis horizontal, and if the optical arrangements are perfect, corresponding points A and A′ on the base and curve lines are on the same generator of the drum whilst the record is being obtained. This implies that after development, when the sheet is flat, AA′ is strictly perpendicular to the base line AH, which

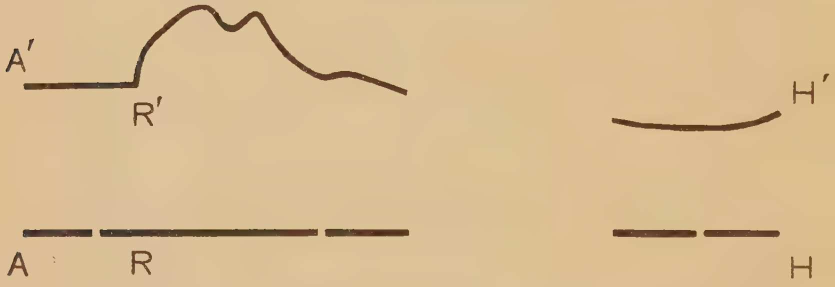

FIG. 1.—THE TWO PARTS OF THE FIGURE REPRESENT THE BEGINNING AND END OF A 24 HOUR RECORD.

represents the successive positions on the paper of the spot of light reflected from the fixed mirror.

The source of light is usually an illuminated slit, vertical in the case of the D and H instruments. The extremities of the image of this slit as formed by an ordinary lens placed between the slit and the two mirrors—which are closely juxtaposed—is or may be cut off by a horizontal slit between the mirrors and the drum ; but the condensation of the image to a point is due to a hemi-cylindrical lens with horizontal axis, a few centimetres in front of the photographic paper. By reducing the width of the vertical slit one may reduce the thickness of the base and curve lines to a fraction of 1 mm. ; but this does not necessarily increase the accuracy of one's time measurements. It is the shape of the spot of light and its length in the direction parallel to the time line that are of primary importance for this purpose.

In the case illustrated by Fig. 1 the time is derived from the breaks in the base line, which represent the intervention of a shutter, actuated by the clockwork, for a short time at successive regular intervals. In the Kew magnetograph the shutter comes on four minutes before each even hour, and goes off at the hour. As a check, the times of starting and stopping each record are noted.

The curves are measured in various ways, representing different degrees of accuracy. At Kew, for instance, the usual practice is to employ glass scales, an idea of which may be derived from Fig. 2. The horizontal line KLNM is intended to be placed immediately over the upper edge of the base line (AH of Fig. 1). The long lines PL, QM . . . have a millimetre scale laid off on them, intended to measure the ordinates of the magnetic curve (A′H′ of Fig. 1). LM and MN represent each an hour of time, and the hour space is subdivided into six 10-minute intervals. If we wish to measure the time of occurrence of a disturbance, say R′ in Fig. 1, we adjust the scale so that R′ lies on one of the vertical lines, say QM, and then observe where the last previous or next subsequent break in the base line comes on the time scale KN. To get times to one minute involves estimating to tenths of the 10-minute divisions shown. In the ordinary Kew pattern instruments one hour is represented by 15 mm., and so one minute by 0·25 mm.

We have thus far assumed—what is, unfortunately, often not the case—that corresponding points on the base and magnetic traces are accurately on a perpendicular to the base line, and that the long lines PL, QM of the glass scale are truly at right angles to KN. If PL and QM are not perpendicular to KN, the error in determining the time of a disturbance increases with the length of the curve ordinate. Other possible sources of error are irregularity in the going of the clock which turns the drum and stretching of the photographic paper. Allowance can, of course, be made for some at least of the errors, but their complete eradication is hardly feasible.

Higher accuracy is obtainable in various ways. Thus, all measurements may be made with a reading microscope capable of reading to 0·01 mm. The time scale may be opened out, and the light interrupted at one-hour or shorter intervals. Eschenhagen magnetographs usually allow 20 mm. to the hour, and the shutter comes on hourly. In some cases the time break is applied to the magnetic curve itself.

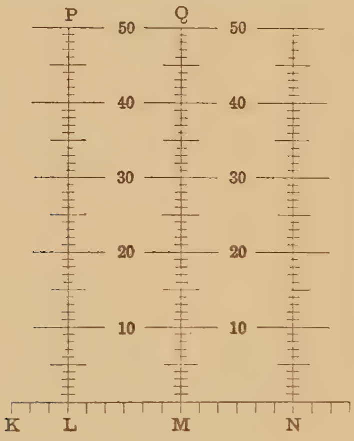

FIG. 2.

For ordinary purposes accuracy to one minute is abundant, and this is all that has usually been aimed at at most British observatories, including Kew.

§ 7. The times for the 15 disturbances at Greenwich, Kew, Falmouth, Helwan and Zi-ka-wei were generally given only to the nearest minute. In one instance at Kew, and in a few instances at Falmouth, they were given to 0·5 minute. At the other stations—with the probable exception of Wilhelmshaven, where the last figure is always 4 or 9—the times appear to be given to 0·1 minute; but I do not suppose that even the most optimistic would claim this order of accuracy. The degree of accuracy actually reached is probably fairly represented by the figures in the first three columns of Table V.

TABLE V.

Disturbance.	Average departure from the arithmetic mean time.			Mean time differences.	
No.	D.	H.	V.	D—H.	V—H.
1	0·93	0·58	1·01	+0·1	0·0
2	1·05	1·08	1·57	+0·9	+1·2
3	0·88	1·12	1·62	+0·4	+1·2
4	0·98	1·17	1·17	−0·3	+0·3
5	0·71	0·80	1·20	+0·1	+0·8
6	0·94	1·17	1·23	−0·2	+0·3
7	0·89	1·10	1·27	+0·7	+0·9
8	0·82	0·74	1·29	+0·1	+0·3
9	0·77	1·10	1·39	−0·3	+1·7
10	0·69	1·01	0·95	+0·1	+0·3
11	0·70	0·85	0·99	0·0	+1·0
12	1·09	1·01	1·27	0·0	+0·8
13	0·94	1·26	2·09	+1·2	+0·2
14	1·06	0·77	0·94	+0·6	+0·2
15	0·94	0·75	1·01	+0·2	+0·5
Mean	0·89	0·97	1·27	+0·24	+0·65

To explain the table, consider the entry 0·93 under D (declination) for disturbance No. 1. This represents the average numerical difference in minutes between the times deduced from the D curves at the several stations and the mean time of occurrence (19 h. 56·6 m.) derived from all the stations combined. The two last columns give the algebraical excesses of the mean times derived from the D and V curves respectively over those derived from the H curves. For instance, the first entry, +0·1, under D-H indicates that the time from the D curves was on the average 0·1 minute later than that given by the H curves. The absolute mean times have been already given in Tables I. and III.

The best agreement between the different stations—and so, presumably, the highest accuracy—is that shown by the H times for disturbance No. 1; but, speaking generally, the agreement is a little closer for the D than the H times. A possible explanation is that there is less variation of type and of scale value in D than in H magnetographs. The disturbances were, as a rule, largest in H, so that, *ceteris paribus*, one would have anticipated the H times to show the best agreement. The agreement is decidedly least good in the case of V, but the inferiority is less marked than I had expected. The H mean time preceded the V mean time on all but one occasion, when they were identical, the average lag in the V time relative to the H being approximately two-thirds of a minute. The mean D time preceded the H on three occasions, but followed on 10, and on the average was almost $\frac{1}{4}$ minute later.

It has, however, to be borne in mind that only a few of the stations gave times from all three elements for all the disturbances, omissions being most common in the case of V, and that four of the stations limited their attention to D and H. Thus, the mean results in Table V are not exactly what we should have got if we had confined ourselves at each station to occasions on which all three elements were represented. The apparent lag in V owed a good deal to two or three stations, including Val Joyeux and Falmouth. It was exhibited, however, to a greater or less extent at most stations. Thus, at Potsdam, one of the few stations which contributed D, H and V data for all the disturbances, the mean apparent lag in V as compared with H was 0·89 minute. The most natural inference would seem to be that disturbances of the type considered have at the start no vertical component. This may be true, but the phenomenon is probably due at least partly to other causes. Disturbances which start from a zero value must, as a rule, take a short time to produce an effect visible on the curve. The disturbances considered were at most stations much larger in H than in V; thus a slight apparent lag in V might be expected, even if the V and H magnetographs were equally sensitive to small changes of force. As a matter of fact, the V instrument is more liable to frictional retardation than the H and D instruments. The question is one which deserves further inquiry.

§ 8. From one point of view the figures in Table V. somewhat exaggerate the inaccuracy of time determinations. The kind of disturbance most appropriate for accurate time measurements consists of a sharp movement on the trace in one direction, preceded by an absolutely quiet time of some duration. Some of the 15 disturbances by no means fulfilled these conditions, and some of the larger differences almost certainly represent misunderstandings as to the precise movement intended to be measured. The occurrence of a few large differences was responsible in most cases for a considerable fraction of the mean differences given in Table V. It thus appeared desirable to make the analysis of the differences presented in Table VI. It shows how many of the differences from the mean at each station lay between 0 and 0·5 minute, between 0·6 and 1 minute, or exceeded 1 minute.

TABLE VI.

Station.	D.				H.				V.			
	0 to 0·5	0·6 to 1·0	> 1·0	Total.	0 to 0·5	0·6 to 1·0	> 1·0	Total.	0 to 0·5	0·6 to 1·0	> 1·0	Total.
Sitka	4	5	4	13	7	1	5	13	5	2	5	12
Ekaterinburg	3	4	5	12	4	5	4	13	2	3	7	12
Rude Skov	3	3	3	9	3	2	3	8	3	1	0	4
Stonyhurst	3	2	5	10	4	2	6	12	0	1	4	5
Wilhelmshaven	4	1	9	14	4	5	4	13	...	...	...	...
Potsdam	11	3	1	15	9	6	0	15	9	4	2	15
De Bilt	3	5	2	10	8	4	0	12	1	2	2	5
Greenwich	7	3	5	15	5	6	4	15	2	5	7	14
Kew	5	5	5	15	7	4	4	15	...	...	...	...
Uccle	4	6	3	13	5	5	5	15	1	2	3	6
Falmouth	4	6	4	14	4	3	8	15	0	3	11	14
Val Joyeux	7	1	5	13	5	2	8	15	1	2	10	13
Munich	8	5	2	15	5	2	7	14	...	...	...	...
Pola	3	0	7	10	1	2	10	13	0	1	6	7
Agincourt	7	4	4	15	7	4	4	15	...	...	...	...
Tortosa	6	0	5	11	3	2	5	10	3	1	7	11
Baldwin	11	4	0	15	11	2	2	15	7	6	2	15
Cheltenham	8	5	2	15	9	3	3	15	6	4	5	15
Zi-ka-wei	0	5	6	11	0	7	7	14	2	2	5	9
Dehra Dun	2	5	5	12	8	4	2	14	4	3	1	8
Helwan	3	2	4	9	2	2	7	11	2	2	5	9
Barrackpore	3	1	2	6	8	3	1	12	3	1	4	8
Honolulu	6	3	6	15	4	4	7	15	4	6	5	15
Toungoo	2	2	1	5	6	2	1	9	1	0	2	3
Bombay	7	3	5	15	9	2	4	15	5	5	5	15
Porto Rico	8	4	2	14	10	3	2	15	6	4	4	14
Kodaikanal	2	0	0	2	6	2	6	14	4	2	2	8
Buitenzorg	7	2	6	15	6	2	7	15	3	3	7	13
Samoa	4	1	3	8	8	2	1	11	1	1	4	6
Pilar	6	3	5	14	3	3	7	13	4	0	9	13
Totals	151	93	116	360	171	96	134	401	79	66	124	269
As percentages of totals	42	26	32	...	43	24	33	...	29	25	46	...

Goodness of agreement with the mean is not a universally satisfactory measure of merit. The winner of a newspaper prize for a list of the 100 best books in the popular estimation may be an exceedingly poor judge of literature. Some allowance may be necessary for this aspect of the case in regard to Table VI. Also, at stations where times are given for only a fraction of the disturbances, it is not improbable that the disturbances omitted included those where identifications were most difficult and large errors most likely.

The agreement with the mean estimate is outstandingly good at Potsdam, the departure from the mean exceeding one minute in only one instance in the case of D and H and two instances in the case of V. Baldwin and Cheltenham, two of the U.S. stations whose times were revised in the light of the Potsdam curves, show also a very superior agreement.

On the whole, in the case of D and H, about two-thirds of the time estimates differed from the mean by not more than a minute, and fully 40 per cent. by not more than half a minute. Greenwich and Kew, though giving times only to the nearest minute, show practically the average closeness of agreement with the mean.

In the case of V the agreement with the mean is decidedly less close than with the other elements. It is, however, quite good at a few stations, all of which showed specially good agreement in the case of D or H.

§ 9. A large average departure from the mean may arise from large irregular errors, or from a prevailing tendency to be late or early in one's estimate. This point was investigated and the results appear in Table VII., which shows the mean numerical and the mean algebraic departure from the accepted time of the disturbance. The plus sign indicates that the estimated time was late.

To explain Table VII., take the case of D at Sitka. The estimated time of commencement on the average of the 13 disturbances measured at Sitka was 0·82 minute in "error." The error was not always in one direction, but the prevailing tendency was to be late, and on the average of the 13 occasions the time was 0·48 minute late. Some stations, including Sitka, Ekaterinburg, Stonyhurst, Greenwich, Uccle, Zi-ka-wei and Bombay, exhibit a tendency in the same direction, whether fast or slow, in all the elements. In the case at least of Greenwich this cannot well be ascribed to systematic clock errors or to any misapprehension as to longitude. Other stations are fast in the times from one element and slow in those from another. Pola and Val Joyeux are outstanding examples. At Pola the V times were all fast, on the average by 2·09 minutes, while the D times were on the average 1·47 minutes slow and were fast only once. Falmouth, again, in the case of D, showed practically no mean algebraic error, while its V times were on the average 1·60 minutes slow. This cannot, moreover, be put down to clock errors, because the D, H and V drums are turned by the same clock.

TABLE VII.—*Numerical and Algebraic Departures from Mean.*

Station.	D.			H.			V.		
	No. of observations.	Average departure from mean.		No. of observations.	Average departure from mean.		No. of observations.	Average departure from mean.	
		Numerical	Algebraic.		Numerical.	Algebraic.		Numerical.	Algebraic.
Sitka	13	0·82	+0·48	13	0·87	+0·73	12	0·77	+0·27
Ekaterinburg	12	0·98	+0·65	13	0·92	+0·78	12	1·37	+1·08
Rude Skov	9	0·74	−0·54	8	0·81	+0·41	4	0·35	+0·05
Stonyhurst	10	1·35	−0·77	12	1·39	−0·74	5	1·52	−1·00
Wilhelmshaven	14	1·36	−1·21	13	0·94	−0·34	...	...	...
Potsdam	15	0·45	−0·12	15	0·44	+0·11	15	0·63	+0·35
De Bilt	10	0·73	−0·03	12	0·42	−0·23	5	0·78	+0·50
Greenwich	15	1·03	+0·36	15	1·24	+0·47	14	1·24	+0·12
Kew	15	0·98	−0·34	15	0·92	−0·07	...	...	...
Uccle	13	0·88	+0·67	15	1·09	+0·38	6	1·02	+0·38
Falmouth	14	0·91	−0·01	15	1·31	+0·47	14	3·36	+1·60
Val Joyeux	13	0·95	+0·06	15	1·34	−0·50	13	2·28	+2·19
Munich	15	0·59	−0·12	14	1·06	+0·06	...	...	...
Pola	10	1·49	+1·47	13	1·59	−1·07	7	2·09	−2·09
Agincourt	15	0·72	+0·07	15	0·83	+0·41	...	...	...
Tortosa	11	0·93	0·00	10	1·37	−0·31	11	1·30	−0·35
Baldwin	15	0·41	+0·01	15	0·54	+0·30	15	0·62	−0·17
Cheltenham	15	0·58	+0·18	15	0·63	+0·41	15	0·88	+0·05
Zi-ka-wei	11	1·63	−0·57	14	1·32	−0·96	9	1·69	−1·11
Dehra	12	0·94	+0·27	13	0·62	+0·25	8	0·60	−0·35
Helwan	9	0·84	+0·84	11	1·39	+0·03	9	1·17	+0·59
Barrackpore	6	0·60	+0·27	12	0·56	+0·22	8	1·06	−0·21
Honolulu	15	0·89	−0·09	15	1·03	−0·68	15	0·94	−0·81
Toungoo	5	0·70	+0·26	9	0·63	+0·03	3	1·37	−1·10
Bombay	15	0·84	−0·67	15	0·60	−0·24	15	0·93	−0·93
Porto Rico	14	0·82	+0·55	15	0·62	+0·11	14	0·97	+0·13
Kodaikanal	2	0·25	−0·25	14	0·93	+0·81	8	0·71	+0·24
Buitenzorg	15	1·09	−0·55	15	1·21	−0·25	13	1·42	−0·76
Samoa	8	1·07	−0·30	11	0·67	−0·18	6	1·28	+0·85
Pilar	14	0·91	−0·11	13	1·67	−0·92	13	1·60	−1·14
Arithmetic means	...	0·88	0·39	...	0·97	0·42	...	1·23	0·71

In cases where a decided tendency was exhibited to depart from the average times in a fixed direction, it would probably be worth while to make some artificial disturbances at definite instants and compare the known times with those derived from measurements of the curves.

The disturbances covered a period of over three years, and the idiosyncrasies presented by some of the stations seemed to persist throughout. Thus, by comparing certain selected pairs of stations, one could obtain evidence seemingly favourable to Dr. Bauer's theory, even when one included a large number of disturbances. If, for instance, we compared Sitka and Bombay times, we should get on the average, from Table VII., as the time to travel between the two stations, 1·15 minutes from the D curves, 0·97 minute from the H curves and 1·20 minutes from the V curves. So, again, for the mean time between Buitenzorg and Greenwich, we should get 0·91 from D, 0·72 from H and 0·83 from V. But in either case we could obviously derive absolutely contradictory results from pairs of intermediate stations.

At least five different patterns of magnetographs were represented: The Mascart and Eschenhagen, with very small magnets; the Watson, with composite systems formed of a number of parallel wires; the Kew, with magnets weighing about 87 grammes; and the Greenwich, which stand to the Kew somewhat as these stand to the Eschenhagen. It would appear that to the degree of accuracy customary in ordinary slow runs no very marked difference existed between the different patterns. If, however, one wished to have comparative times on quick-run curves correct to within a few seconds, it would be advisable, if not absolutely necessary, to employ instruments of the same pattern, similarly sensitive.

ABSTRACT.

The Paper is a sequel to one read before the Society in November, 1910 (" Proc.," Vol. XXIII., p. 49), dealing with the times of commencement of 15 magnetic disturbances discussed by Mr. R. L. Faris, and supposed by him to support Dr. L. A. Bauer's theory that the commencing movements of magnetic storms travel round the globe at rates of the order of 100 km. per second. In his previous Paper the author criticised Mr. Faris' Paper, and suggested that for an adequate test of Dr. Bauer's theory data could only be obtained from a number of stations encircling the earth. Shortly afterwards Dr. Bauer issued a circular requesting magnetic observatories to send him their measurements of the times of commencement of the 15 magnetic storms discussed by Mr. Faris. Upwards of 30 stations sent in data in answer to Dr. Bauer's request. A discussion of the data derived from the horizontal force curves has recently been published by Dr. Angenheister, whose conclusions are unfavourable to Dr. Bauer's theory. The present Paper deals with the data from the

declination and vertical force curves as well as those from the horizontal force curves following a somewhat different method from that adopted by Dr. Angenheister. The bearing of the data on Dr. Bauer's theory is discussed, and the peculiarities of the individual stations are considered.

DISCUSSION.

Prof. S. P. THOMPSON thought that Dr. Chree's conclusions pointed to the necessity of equipping all magnetic observatories with similar instruments and making them keep records to a tenth of a second.

Mr. R. S. WHIPPLE mentioned the enormous amount of work involved in the reduction of these observations. The instrumental errors due to backlash and other causes were apt to be so great that it was really marvellous that observers went so far as to discuss times of the magnitude involved. He emphasised the need of standardised instruments.

Prof. C. H. LEES said that, like most people who had read Dr. Bauer's Paper when it appeared, he had had grave doubts of the instrumental accuracy. He thought the method of calculating the results rather biassed. He had set a student to work out the correlation number of the times of the disturbances and the longitudes of the observatories. The number was too small to justify any conclusion as to connection between the quantities.

Mr. C. W. S. CRAWLEY thought it would be an advantage to work on a much larger time scale, say, 2 in. or 3 in. per minute, and make use of the little everyday disturbances. He had observed a large number of these in this way. Sometimes the disturbance commenced with a contrary kick and sometimes it did not. The phenomenon, when present, was quite easy to detect. He asked Dr. Chree if no connection ever existed between disturbances and volcanic action.

Dr. A. RUSSELL asked the author if it was known whether atmospheric gales had any effect in producing magnetic storms. It was well known that in the Arctic and Antarctic regions violent magnetic storms were of frequent occurrence. These were often attributed to swarms of electrons flying past the earth. If this were so it would be possible for vortex rings of electric current to be set up which would travel comparatively slowly round the surface of the globe. These might conceivably alter the force components of terrestrial magnetism. In his opinion, the possible effects of the currents existing in the interior of the earth, or set up on its surface by celestial disturbances, ought to be considered as well as the currents in the upper atmosphere.

The AUTHOR, in reply, stated that the objection to using a wide time-scale was that an enormous amount of photographic paper would be necessary. Moreover, if the scales are too open, the curves present such a large number of features demanding attention that the staff usually at the disposal of an observatory would be unable to cope with the work involved. The objection to using the small disturbances, which were quite frequent, was the difficulty of ensuring that all the observers were dealing with the same disturbance. Dr. Moos had suggested a connection between certain magnetic disturbances and volcanic action, but he did not think the evidence was conclusive. Dr. Leyst, of Moscow, had established a relation between the amplitude of the ordinary magnetic variations and the height of the barometer.

XVII. *A Lecture Experiment on the Irrationality of Dispersion. By Prof.* S. P. Thompson.

Newton's method of crossed prisms throws an oblique spectrum on the wall. If the prisms used are of identical kinds of glass the oblique spectrum is straight from red to violet. But if different kinds of glass are used, the spectrum is curved by reason of the irrationality of dispersion in one or both of the glasses. If a diffraction grating is used instead of one of the prisms, then the curvature which is observed is that resulting from the irrationality of dispersion of the particular prism employed ; and for all known kinds of glass the refraction for blue and violet rays is disproportionately large. To exhibit these effects in the lecture theatre a diffraction grating of 12,000 lines to the inch is employed to cast on the screen a horizontal spectrum of the first order, the light from an arc lamp being sent through a small round or square hole. On interposing a prism to disperse the light vertically upwards, the resultant oblique spectrum is finely curved, being concave upwards. If in the arc lamp the carbons used are those supplied for commercial " flame arcs," the effect is more brilliant, since the spectrum contains several bright bands. If an alloy of zinc, thallium and lithium is introduced into the crater of the positive carbon, a still more striking series of bright points is produced in the curved spectrum.

XVIII. *On the Ratio of the Specific Heats of Air, Hydrogen, Carbon Dioxide and Nitrous Oxide.* By H. N. Mercer, *B.Sc., Research Student at the Imperial College of Science and Technology, London, S.W.*

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The main object of the following experiments was to test the accuracy with which the ratio of the specific heats could be measured, employing small quantities of the gas to be tested, with the ultimate view of performing experiments on the variation of the ratio at different temperatures. The method employed was to observe, with a platinum thermometer of very fine wire, the instantaneous fall of temperature corresponding to a given rapid fall of pressure. This method was first employed by Lummer and Pringsheim ("Smithsonian Contributions," 1898), using a bolometer strip 0·001 mm. thick, with a vessel of 50-100 litres capacity. It was subsequently employed by Makower ("Phil. Mag.," February, 1903) for the determination of the ratio of the specific heats of steam, with modifications introduced by Prof. Callendar, which consisted in the substitution of a platinum thermometer, compensated for conduction along the leads, in place of the bolometer strip, and in the adoption of an automatic contact for closing the galvanometer circuit at the right moment, which made it possible to work with smaller quantities of gas. The vessel employed by Makower was about 9 litres in capacity, but his experiments showed that a much smaller vessel might have been used. The apparatus employed in the present investigation was similar to that used by Makower, but it was found that with due precautions an equal degree of accuracy was obtainable with a vessel of only 300 cubic centimetres capacity.

The advantage of the method, as compared with that of Clement and Desormes, is that the adiabatic fall of temperature observed at the centre of the vessel is little affected by the walls, and that it is unnecessary to close the vessel immediately after making an expansion. As compared with methods depending on the velocity of sound there is the further advantage that whereas an error of 1 per cent. in the velocity involves an error of 2 per cent. in the ratio of the specific heats, an error of 1 per cent. in the observed fall of temperature produces an error of less than $\frac{1}{2}$ of 1 per cent. in the ratio.

Assuming that the gas obeys the law $pv=RT$, and that its specific heat S at constant pressure may be regarded as constant over the small range of temperature of the experiment, the expression for the entropy is

$$S \log T = R \log p + \text{constant}.$$

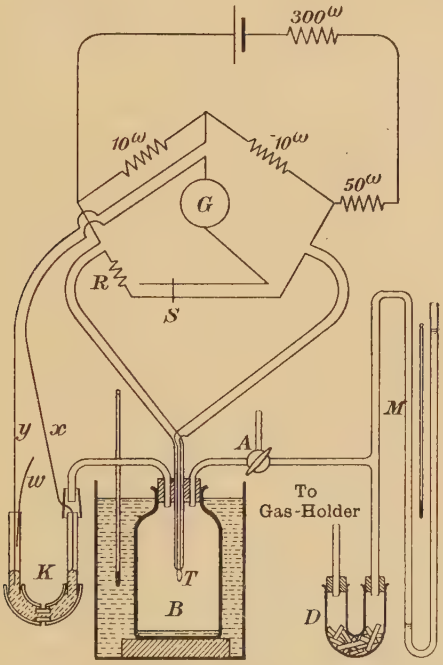
If the gas is suddenly expanded under isentropic conditions from the state p_1T_1 to the state p_2T_2, we obtain the simple relation

$$S/R = (\log p_1/p_2)/(\log T_1/T_2),$$

from which the value of S may be directly deduced, since R is known, or the ratio of the specific heats γ, which is equal to $S/(S-R)$. It would be in many cases easier to determine the value of S in this way than by a direct calorimetric determination, and the value found would not, as a rule, be greatly, if at all, inferior in accuracy.

If any other form of characteristic equation is assumed, the value of the ratio may be deduced in a similar manner from the expression for the entropy. But this has not been done in the present Paper, owing to the great variety of the expressions which have been proposed to represent the small deviations of gases from the ideal state. The corrections thus introduced would be small in most cases, but would differ greatly according to the type of equation assumed. It is possible, for instance, to have a considerable deviation from Boyle's law, without any variation in γ, provided that the ratio E/pv is equal to $(S-R)/R$.

The apparatus consisted of a bottle, B, of some 300 cubic centimetres capacity containing the gas to be experimented upon. Near the centre of the bottle was fixed the platinum thermometer T; as this should be able to follow closely the variation of the temperature of the surrounding gas, the type used was the same as employed by Prof. Callendar in his steam-engine experiments, and consists of two loops of fine platinum wire, one being much shorter than the other. With this type the end-effect error due to conduction along the leads is eliminated. At A was a three-way tap by which gas could be admitted from the gas-holder (not shown in figure) to the bottle, or communication opened between the bottle and the outer air. Expansion was thus allowed to take place from a pressure, p_1, some 50 mm. of mercury above atmospheric down to atmospheric pressure p_2, this latter being obtained by reading the barometer. An oil-gauge, M, was used in the pressure measurements. A drying tube, D, was inserted between the gas-holder and bottle B, and in addition the bottle contained a little strong sulphuric acid. A mercury key, K, also communicated with the bottle through a short, narrow tube, and served as an automatic contact closing the galvanometer circuit shortly after the expansion had taken place. The wire w, together with the wires x and y, were used in the chronograph observations.



Temperature Measurements.—The point of balance on the bridge wire corresponding to the steady temperature T_1 was found, and the time noted when this was done. Communication was opened between the bottle and the gas-holder and time allowed for the temperature of the gas in the bottle and the oil in the manometer each to become steady. The sliding contact S on the bridge was adjusted by judgment from previous trials nearly to a position where there would be no current through the galvanometer when the circuit was completed by the mercury key after the expansion. If the contact S was at the exact point of balance for the instantaneous temperature T_2, the spot of light on the galvanometer scale remained stationary for an instant and then moved off to the left as the gas heated up to regain the ordinary temperature. It was found advisable to make the adjustment of S so that a small throw to the right was obtained, the value of a small throw having been determined in terms of a shift of 1 mm. on the bridge wire. The exact point of balance could then be estimated. Having made the expansion and observed the throw, the galvanometer was brought to rest, and the point of balance for the steady temperature determined as early as possible. As the steady temperature T_1 was found to vary slightly during a series of observations, the time was noted at which the above readings were taken, so that the value of T_1 corresponding to the time at which T_2 was measured could be estimated.

Readings on the bridge wire could be taken with an accuracy to give temperature to 0·01°C.

The speed of the mercury key K was adjusted until a maximum fall of temperature was indicated for a given difference of pressure (p_1-p_2). This was done either by means of a pinch-cock on the rubber tube connecting the two limbs of the key or by inserting a glass tube with capillary bore to act as throttle.

The time which elapsed between the opening of tap A to the outer air and the closing of the galvanometer circuit by the mercury key was measured with the aid of a chronograph. For this purpose the wires w, x and y were made use of; wire w just made contact with the mercury at the pressure p_1, x at the atmospheric pressure, and y always made contact with the mercury. The galvanometer being disconnected, the key K was placed in series with one pen of the chronograph and the two storage cells, y being used as one terminal of the key and x and w together as the other. When an expansion took place the pen gave one kick when contact was broken at the end of wire w and another kick when contact was made on wire x. The other pen of the chronograph ticked off seconds.

In exactly the same way, by replacing the capillary throttle in the mercury key by an open tube, the time taken by the gas to expand from p_1 to p_2 was measured.

Pressure Measurements.—The manometer M used to measure the differences of pressure (p_1-p_2) consisted of a U-tube graduated in millimetres on both limbs and containing machine oil. The oil being very viscous, it was important that no great motion of the oil should take place. This condition was secured by the use of the gas-holder (not shown in the figure), having a volume of about 4 cubic ft., and with which the oil gauge M always communicated. Under these circumstances the motion of the oil for each experiment was only about 2 mm., so that little time was lost in waiting for the oil to become stationary.

The density of the oil was carefully determined at ordinary temperatures by comparison with a water manometer and its coefficient of expansion measured. If h_0 be the head in centimetres of mercury at 0°C., corresponding to a head h in centimetres of oil at t°C., the expression for h is

$$h_0=h(0{\cdot}0669)[1-0{\cdot}00068\ (t-18{\cdot}8)].$$

Readings of the gauge were taken immediately before making an expansion.

Preliminary Determinations.—The bridge wire was calibrated, but in no case was the correction greater than 0·01 centimetre. As 0·01 centimetre was the limit of accuracy in the observation of a bridge reading, no account was taken of the corrections. The bridge resistances R were calibrated in terms of the bridge wire.

Standardisation of the Platinum Thermometer.—For this purpose a mercury thermometer was placed near the platinum thermometer inside the bottle, and this mercury thermometer afterwards compared with a standard mercury thermometer. The resistance of the platinum thermometer is given in terms of the resistance of 1 centimetre of bridge wire as unit.

——	Mercury thermometer reading.	Resistance of platinum thermometer.
Bottle immersed in melting ice…………	0·75°C.	638·44
Bottle immersed in water at room temperature ………………………………	17·16°C.	655·57
Difference ………………………	16·41°C.	17·13

Hence 17·13 centimetres correspond to 16·41°C., therefore a shift on the bridge wire of 1 centimetre corresponds to a change in temperature of 0·9579 deg.

The heating effect of the current on the fine wire of the thermometer was measured and found to be 0·14 deg., but no account was taken of it because it did not vary appreciably during the tests.

The following is a typical series of observations with air :—

—	Time.	Manometer readings.	Difference p_1-p_2 cms. Oil.	Steady point of balance corresponding to T_1.		Point of adjustment for temperature T_2.	Throw. Scale divisions.	Estimat point o balance correspon ing to T
				Observed.	Estimated.			
	3·2	...	...	32·89	...	...	...	...
I.	3·4	0·99 80·84	79·85	...	32·90	27·34	3	27·31
	3·5	...	...	32·90	...	...	...	...
II.	3·8	1·10 80·73	79·63	...	32·91	27·38	2	27·36
	3·10	...	...	32·92	...	...	...	...
III.	3·13	1·19 80·68	79·49	...	32·94	27·44	4	27·40
	3·14	...	...	32·94	...	...	...	...

—	p_1-p_2 mm. mercury.	p_2 mm.	p_1 mm.	Shift on bridge wire corresponding to T_1-T_2. cms.	T_1-T_2	T_1 °Abs.	T_2 °Abs.	Ratio.
I.	53·37	764·20	817·57	5·59	5·48	292·04	286·56	1·3900
II.	53·22	764·20	817·42	5·55	5·44	292·05	286·61	1·3875
III.	53·13	764·20	817·33	5·54	5·43	292·08	286·65	1·3873

About 50 observations of this kind were taken with air, 10 with hydrogen, 30 with carbon dioxide and 30 with nitrous oxide. The carbon dioxide was obtained fairly pure by using the last half of a large bottle of liquid. The residual impurity was oxygen. The gas was tested by agitation with caustic potash solution when about 98 per cent. was absorbed. With pyrogallol about 2 per cent. was absorbed. In the case of nitrous oxide when subjected to the ordinary gas analysis about 2·6 per cent. was found to be oxygen. Correction was made assuming that the oxygen was present as an impurity of air—that is, about 13 per cent. of air. The following approximate formula was used to make the correction :—

$$\gamma \text{ for mixture} = 0{\cdot}87\gamma \text{ for nitrous oxide} + 0{\cdot}13\gamma \text{ for air.}$$

The mean values of the ratio obtained in the first instance were as follow :—

Air	1·382	
Hydrogen	1·397	
Carbon dioxide	1·277	(corrected for an impurity of oxygen).
Nitrous oxide	1·229	(corrected for an impurity of air).

Corrections.—There are two corrections to be applied. (1) In the measurement of T_2 the thermometer will be at a higher temperature than the surrounding gas, due to radiation from the walls of the vessel. As it was thought that during the first series of experiments the platinum thermometer was not very clean it was removed and cleaned by passing it several times through a spirit flame. The thermometer was then replaced, re-standardised and a new series of values taken with air.

Value of γ for air with bright thermometer	1·390
Showing an increase of	0·008
Equivalent to an increase in $T_1 - T_2$ of	0·079°C.

The corrections to $T_1 - T_2$ for the other gases compared with that for air will be in the inverse ratio of the conductivities.

Gas.	Correction to $T_1 - T_2$.	Correction to γ.	Values corrected to bright thermometer.
Air	...	...	1·390
Hydrogen	0·011	0·0011	1·398
Carbon dioxide...	0·134	0·0119	1·289
Nitrous oxide	0·118	0·0105	1·239

The error due to radiation for the bright thermometer was determined by performing experiments with the platinum thermometer coated with platinum black, other conditions being the same. Making use of a determination due to Lummer and Pringsheim—namely, that the absorption by a platinum-blacked surface is 15 times as great as that of a bright surface —the error due to radiation can be estimated. The value of γ obtained for air with the platinum-blacked thermometer was 1·364. Since the value with the bright thermometer was 1·390 the correction to be applied in the case of air is

$$\frac{1{\cdot}390 - 1{\cdot}364}{14} = 0{\cdot}0018.$$

This is equivalent to an increase in $T_1 — T_2$ of 0·0178. As before the correction for the other gases compared with that for air will be inversely as the conductivities.

Gas.	Correction to $T_1 - T_2$.	Correction to γ.	Value corrected for radiation.
Air	0·0178	0·0018	1·392
Hydrogen	0·0025	0·0002	1·398
Carbon dioxide.........	0·0303	0·0027	1·292
Nitrous oxide	0·0265	0·0024	1·241

(2) An error is introduced due to the effects on the thermometer of conduction and convection in the gas during the time which elapses between the expansion taking place and the closing of the galvanometer circuit.

This correction was estimated by repeating observations with another bottle having practically twice the linear dimensions. The dimensions of the two bottles were : Small bottle, internal diameter 6·2 cm., height 10·1 cm. ; larger bottle, internal diameter 11·6 cm., height 19·5 cm. When the correction is small it may be taken to vary inversely as the linear dimensions, so that the correction for the larger bottle is approximately equal to the difference between the values obtained with the two bottles.

The values obtained and the corrected values for air and nitrous oxide are given in the table :—

Gas.	Values corrected for radiation.		Correction for larger.	Final value.
	Small bottle.	Larger bottle.		
Air	1·392	1·396	0·004	1·400
Hydrogen	1·398	...	...	...
Carbon dioxide	1·292	1·286	...	...
Nitrous oxide	1·241	1·251	0·010	1·261

The correction in the case of nitrous oxide is somewhat doubtful owing to impurity.

Chronograph Observations.

Gas.	Time of expansion.	Time of mercury key.
	Seconds.	Seconds.
Air, small bottle	0·16	0·23
Air, larger bottle	0·77	0·97
Carbon dioxide, larger bottle	0·78	1·00
Nitrous oxide, larger bottle	0·49	0·79

The quantity in the last column is the time which elapsed between the opening of the tap A to the outer air and the closing of the galvanometer circuit.

Conclusion.—It is interesting to compare the values thus obtained with direct measurements of the specific heats. Taking the value of R per gramme of air to be 0·06866 calorie per degree, the value of the specific heat S from the present experiments comes out 0·2403 calorie per degree, which agrees very closely with Swann's determination at the atmospheric temperature and pressure. The values for the other gases tested are similarly compared in the following table :—

Gas.	Ratio.	R.	S.	Values found by other observers.
Air	1·400	0·06866	0·2403	Swann... 0·2417 Regnault... 0·2374 Holborn 0·2366 Witkowski 0·2372
Steam*	1·303	0·1105	0·475	Regnault 0·4805 Holborn... 0·4652
Hydrogen....	1·398	0·9880	3·4704	Regnault 3·4046 Lussana... 3·402
Carbon dioxide	1·292	0·04521	0·2000	Swann...0·2020 Regnault... 0·2164 Holborn 0·2010
Nitrous oxide	1·261	0·04523	0·2185	Regnault 0·2238 Wiedemann 0·213

Holborn's values are generally admitted to be low, but the agreement is otherwise very good, although for an accurate comparison some small corrections might be required on account of the deviations from the ideal state.

My best thanks are due to Prof. Callendar for his invaluable advice throughout the work.

ABSTRACT.

The main object of the experiments was to test the accuracy with which γ could be measured, employing small quantities of the gas, with the ultimate view of experiments on the variation of γ with temperature. The method employed was to observe with a platinum thermometer of very fine wire the instantaneous fall of temperature corresponding to a given rapid fall of pressure. The method was first employed by Lummer and Pringsheim ("Smithsonian Contributions," 1898), using a bolometer strip, with a vessel of 50-100 litres capacity. It was later employed by Makower ("Phil. Mag.," February, 1903) for the determination of γ for steam, with modifications introduced by Prof. Callendar, which consisted in the substitution of a platinum thermometer, compensated for conduction along the leads, in place of the bolometer strip, and in the adoption of an automatic contact for closing the galvanometer circuit at the right moment, which made it possible to work with smaller quantities of gas. The vessel used by Makower was about 9 litres in capacity. The apparatus employed in the present experiments was similar to that used by Makower, but it was found that with due precautions an equal degree of accuracy was obtainable with a vessel of only 300 cubic cm. capacity.

A table is given in the Paper showing the values of the specific heat at constant pressure for the various gases as calculated from the present experiments. The values show good agreement with direct calorimetric determinations.

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* Makower by the same method. He finds 1·399 for air.

DISCUSSION.

Prof. C. H. Lees thought the Paper marked an advance in the subject. Hydrogen was always a severe test in such measurements, and the corrections were troublesome.

Mr. F. E. Smith asked what was the diameter of the platinum wire used, as there was probably a lag between its temperature and that of the gas. He thought more information was required on the time of making the galvanometer contact. It was possible that the gas might have commenced to cool, and in consequence to lower the temperature of the thermometer, before the correction due to the heating current had attained its final value.

The Author stated that the wire was very fine, about a fortieth of a millimetre in diameter, and he did not think there could be an appreciable lag.

XIX. *A New Type of Thermogalvanometer.* *By* F. W. Jordan, *A.R.C.S.*, *B.Sc.*

Received January 27, 1914.

It occurred to me that the puff of air from an orifice in a chamber when the air within is suddenly heated might be utilised to deflect a small vane. A puff of air can be generated thermally by the passage of a current through a fine wire, by the incidence of radiant heat upon a thin absorbing disc and in a variety of other ways. In the following description I have confined myself chiefly to the development of a suitable thermogalvanometer.

The passage of a current through a heater of small thermal capacity generates first of all a condensational wave in the air or fluid about the wire, as in the hot-wire telephone, and a subsequent slower expansion of the fluid during the period of attainment of temperature equilibrium between the heater and its surroundings. Some idea of the magnitude of the momentum of the puff of air can be obtained from a simple calculation. The absorption of 10^{-6} calories by the air will cause an expansion of 10^{-2} cubic mm. at constant pressure, or if the air be confined at atmospheric pressure in a vessel of volume equal to 1 c.c., the change of pressure would be 10 dynes per square centimetre approximately. It is obvious that for such a small absorption of heat the impulse delivered to a vane in the region of the orifice would be exceedingly small, and in order to make it detectable the controlling torque on the vane must be made very weak. The latter is practicable owing to the fact that the vane and its attachments can be made very light and of small inertia. The whole weight of the suspended system need not exceed 10 mgm. and quartz fibres down to 0·002 mm. diameter can be safely used to control the motion of the vane. The disturbing effect of sudden small variations of atmospheric pressure have been eliminated by a compensation method. The vane is suspended symmetrically in front of two orifices connected with equal chambers, so that the pulsations of atmospheric pressure balance each other in their effects on the halves of the vane. The instrument can be shielded sufficiently from external sources of heat by (1) making the walls of the chambers thick and of good thermally conducting material like copper or brass, and (2) by establishing a good thermal connection between the walls of the two chambers. Under these conditions any absorption of heat will be almost equally divided between the walls of the chambers, and, owing to their large thermal capacities, the rise of temperature of the air in each will be very slow.

Fig. 1 is a horizontal section through the chambers A, B and the vane V. The cylindrical chambers A, B were bored out of the same solid brass cylinder and hermetically sealed at one end by the cover plate C. Each chamber communicated with the half of a tube, D, which was partitioned longitudinally by a brass strip, E. The circular vane V, 2·6 mm. in diameter, was punched out of a thin sheet of aluminium and attached to a fine glass stem. A plane mirror, 2·5 by 4·7 by 0·16 mm., was fastened to the stem beneath the vane. The stem was suspended by a fine quartz fibre, 5·5 cm. long and 0·0025 mm. in diameter, from a torsion head in the upper part of the compartment F. The free period of the suspended system was eight seconds, and the amplitude of vibration became imperceptible after the fourth vibration. The position of the vane V could be observed through the window G and was adjusted so that the vane was quite close and transverse to the partition E. The clearance between the edge of the vane and the tube D was about 0·17 mm. The vane was finally adjusted with the levelling screws so that it was insensitive to an expansion

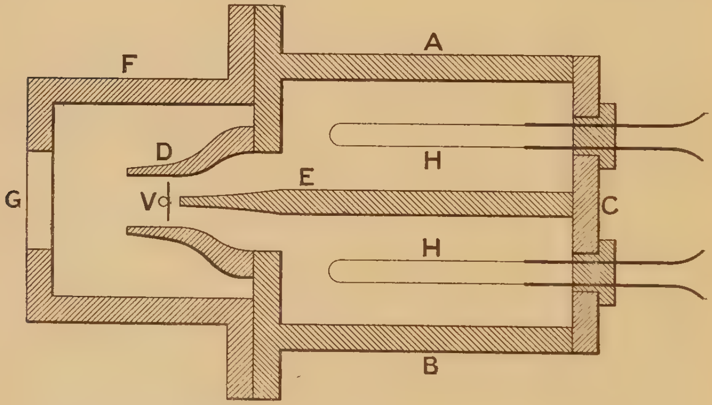

Fig. 1.

of the air caused by touching the compartment F with the warm hand. A heater, H, consisting of a single loop of constantan wire, 0·015 mm. in diameter, was fitted axially in each chamber and their resistances were made approximately equal.

The passage of a current through a heater caused the vane to move to its extreme position in less than two seconds, followed by a return to zero in a time varying from 20 to 5 seconds, according to the clearance between the face of the vane and the edge of the partition E. The distance between the vane and the edge of the partition E was, in the latter case, 0·24 mm., and the sensibility was about 60 per cent. of its value when the vane was quite close to E. The throw of the vane is approximately doubled by diverting the current quickly from one heater to the other, since the inrush through one and the outrush of air through the other orifice both contribute to deflect the vane in the same direction. The instrument was calibrated by diverting a measured current from one heater to the other, and the following table shows how the throw depends on the current :—

Current $C \times 10^3$ amp.	Throw of vane.		Mean throw d.	$\frac{C^2}{d}$.
	Left.	Right.		
0·77	3·40 cm.	3·45 cm.	3·42 cm.	1,736
1·025	5·95 cm.	6·05 cm.	6·00 cm.	1,754
1·230	8·49 cm.	8·67 cm.	8·58 cm.	1,760
1·436	11·53 cm.	11·91 cm.	11·72 cm.	1,755
1·693	16·38 cm.	17·06 cm.	16·72 cm.	1,714

The distance of the scale from the mirror was 64 cm. and the current was measured to about 1 part in 300 with a Weston millivoltmeter. The results indicate that, over a range of 250 mm. on a scale at a distance of 1 metre from the mirror, the throw of the vane is very nearly proportional to the square of the current. The inequality of the throws to the left and right is explained by some slight want of symmetry. These measurements were made at a time when there was very little vibration in the building and the zero did not vary by more than 0·2 mm. from its mean position. The resistances of the heaters were 90·6 and 95·8 ohms, and the sensibility, as deduced from the foregoing figures, was 0·96 mm. per microwatt.

The following table shows how the sensibility varies with the clearance between the face of the vane V and the edge of the partition E. This change of sensibility is due to a short-circuiting of the air current between the orifices :—

Clearance	0·08	0·16	0·24	0·31	0·63 mm.
Sensibility	0·89	0·72	0·53	0·40	0·13 mm. per 10^{-6} watt
Time of return to zero	17	10	5·5	(Undamped) seconds.	

These results also give some idea of how the sensibility would depend on the clearance around the edge of the vane. I find that there is not much to be gained by reducing the size of vane unless the clearance is diminished to a troublesome small value.

It was found in the preliminary experiments that the sensibility varied considerably with the distance between the wire filament and the walls of the chamber. To elucidate approximately the nature of this dependence brass hollow cylinders of various internal diameters were fitted inside one of the cylindrical chambers. The heater consisted of a narrow loop of 0·05 mm. constantan wire, and was arranged so that the loop was centrally situated within the hollow cylinder. The following results indicate how the sensibility depends on the distance between the heater and the walls of the chamber :—

Diameter of chamber	6·0	6·8	7·9	9·5	12·2 mm.
Sensibility	23·5	30·0	40·0	52·5	78

These results show that if the loop was made indefinitely narrow the sensibility would be approximately proportional to the diameter of the chamber. The change of sensibility is obviously due to loss of absorption of heat by the gas owing to its comparatively rapid conduction to the walls of the chamber.

The effect of altering the volume of the chamber was determined by attaching the heater to a closely fitting cylindrical plug within the chamber. The latter was adjusted to several positions, and no appreciable change of sensibility was detected. The available momentum of the puff of air thus seems to be independent of the volume of the chamber, and this is what is to be expected if the expansion of the air is very rapid. It is advisable to keep the diameter of the chamber within certain limits in order to minimise the effects of stray heat and pulsations of atmospheric pressure.

A modification of the above instrument, in which a pair of fine nozzles was substituted for the partitioned tube, has also been tried. The internal diameter of the air chambers was 2·5 mm., or twice that in the above instrument, and the length was increased from 3 cm. to 5 cm. Each of the nozzles was 0·4 mm. in diameter and they were fixed at a distance of 2 mm. apart. The mirror attached to the suspended system was made smaller, and the free period of swing was about six seconds. A calculation showed that the controlling torque was about 0·6 of its value in the above instrument. The vane was brought close to the nozzles and adjusted in the manner described above. The vane, when very close to the nozzles, developed a troublesome instability which became accentuated by a slight vibration of the instrument. The sensibility in such a position was about 8 mm. per microwatt. When the distance between the vane and the nozzle was increased to 0·3 mm. the instability for moderate throws disappeared, the zero was much steadier and the sensibility was 4 mm. per microwatt. In this type of instrument a movement of the vane from its zero position destroys compensation for external sources of disturbance, and the effect of these becomes added to the puff of air in moving the vane. To free the vane from this objection, it can be made cylindrical and suspended so that the axis of suspension coincides as nearly as possible with the axis *o* of the vane *v* in Fig. 2. The clearance between the nozzles *a*, *b* and the vane will then remain nearly constant during the motion of the vane, and the instability can be much reduced. An alternative method is to arrange a pair of nozzles to deliver a tangential puff of air against a horizontal vane, *v*, as in Fig. 3.

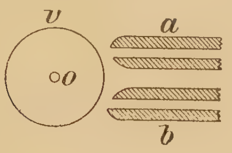

Fig. 2.

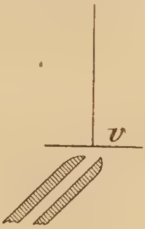

Fig. 3.

The heaters R, R in the above instruments can be polarised by a continuous current C, Fig. 4, as in the Irwin hot wire oscillograph, and then the instrument can be modified for the detection of a continuous or alternate current *c*. If, for example, R=30 ohms, and the polarising current C=0·25 ampere, a continuous current $c'=10^{-6}$ amperes will cause a change in the rate of emission of heat in each heater of $2Cc'R$, or 15 microwatts—a change that can easily be detected with a sensitive instrument.

To test the adaptability of the instrument to the detection of an alternate current, a vane and mirror were fastened to a stretched strip of phosphor bronze, and the period was reduced to 0·033 second. The controlling torque per radian deflection of the vane was about 1·3 dyne cm., so that the period, if necessary, could be diminished considerably by a reduction in the size of the mirror and vane. Two fine platinum wire heaters were mounted in the chambers and polarised by a current of 0·25 ampere. Under these conditions the passage of a current, c', of 0·5 milliampere through the heaters produced an oscillation of 1 mm. on a scale at a distance of 1 metre from the mirror. The vane would, therefore, oscillate under the action of a superposed alternate current c of moderate frequency. I have not yet made any observations on the wave form given with this instrument during the passage of an alternate current for the exact nature of the dependence of the deflection on the current at any instant is not at all obvious.

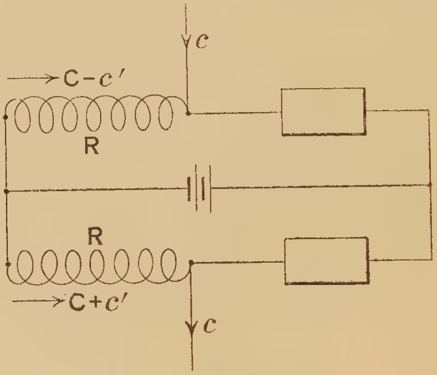

FIG. 4.

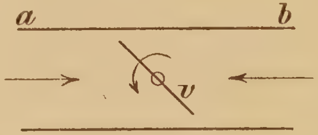

FIG. 5.

A steady deflection of the vane can be obtained with alternate current by making a slight modification in the adjustment of the vane. A plane vane, *v*, Fig. 5, is suspended so that it is initially inclined at an angle of 45 deg. with the axis of the channel connecting the chambers A, B. If the axis of rotation passes centrally through the vane, then a puff of air in either direction will rotate the vane in the anti-clockwise direction. Thus an alternate current through a fine wire heater in one of the chambers or through two polarised heaters will deflect the vane to a steady position provided that the heated wire is fine enough to follow the variations of the alternate current.

The results of the work with these instruments indicate that, if the use of a fine quartz fibre is not objectionable, a fairly quick and sensitive thermogalvanometer can be constructed on this simple principle. The best dimensions of the instrument can only be arrived at by a series of experiments, and I hope to be able to state more exactly, as a result of future experiments, its possibilities and limitations in its various applications.

South-Western Polytechnic,
Chelsea, S.W.

ABSTRACT.

The puff of air from an orifice in an air chamber when the air within is suddenly heated is utilised in this instrument to deflect a small suspended vane. The current to be measured is made or broken through a heater of small thermal capacity in the air chamber and the outrush or inrush of air through the orifice delivers an impulse to the vane. The disturbing effects of extraneous heat and pulsations of external pressure are eliminated by a compensation method. In one instrument of this type the sensibility was 4 mm. per microwatt and the extremity of the throw of the vane was attained in 2 seconds.

DISCUSSION.

Dr. W. H. **Eccles** said a good deal of work had been done with convection galvanometers, but Mr. Jordan was the first to measure the pulses produced by suddenly heated filaments. About nine years ago he had made some small instruments consisting of a fine filament connected to heavier leads and mounted in a small glass tube suitable for insertion in the ear. If an interrupted current passed through the filament the observer could hear every pulse produced. The instruments varied in sensitiveness. One which had a platinum filament of 12 ohms resistance could detect 0·0025 ampere, while another, with a platinised quartz filament of 900 ohms, could detect 0·00003 ampere.

Mr. W. **Duddell** thought the instrument was very ingenious. He asked if the sensitiveness could be increased to measure quantities of energy of the order of a microwatt.

Mr. **Jordan**, in reply, stated that the fibre was already extremely fine as the inertia was small. He thought the vane could be reduced considerably and the sensitiveness thereby increased without a serious increase in the time of swing.

XX. *An Instrument for Recording Pressure Variations due to Explosions in Tubes.* *By* J. D. MORGAN.

RECEIVED MARCH 10, 1914.

THE object of this Paper is to describe a mechanical oscillograph for recording the pressure variations which accompany a gas or other explosion in an open tube. Primarily the instrument was designed for the investigation of certain phenomena associated with dust explosions, the purpose being to obtain by simple means an accurate diagram of the pressure variations which occur at a given position in a tube in consequence of an explosion.

The conditions with which such an instrument must comply are : (*a*) It must be capable of operating at a high speed under small as well as under comparatively large pressures and (*b*) it must be dead-beat. Further conditions necessitated by the particular work for which it was required are (*c*) it must be robust and not liable to obscure changes and (*d*) it must be capable of producing a large number of diagrams in a short time at negligible cost.

Obviously the best known device which complies with the above conditions is a thin steel or other light elastic metal diaphragm arranged in conjunction with a style for recording the movements of the diaphragm on a revolving strip of smoked paper. Experience proves, however, that such a combination is objectionable for a variety of reasons. The amplitude of the movement of the diaphragm is not proportional to the pressure, and in consequence the diagrams are difficult to interpret. Owing to the complexity of the diagrams it is not practicable to re-draw them by hand to a scale in which the ordinates are proportional to the pressure. Further, the motion of the diaphragm is so small that a long style is required to produce the magnification necessary for a diagram of convenient size, and with such a style it is difficult to make the instrument deadbeat and to avoid defects in the diagram due to bending of the style.

Instead of a diaphragm the author has employed a light steel vane of rectangular form. This is mounted parallel to the explosion tube in a cell presenting a lateral opening to the tube interior. Along three edges the vane is free and along the fourth edge it is attached to a torsion wire. The vane is made

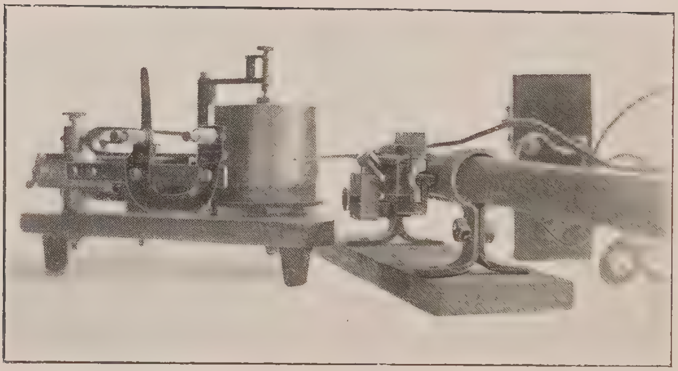

FIG. 1.

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to fit the cell as closely as possible around its edges without touching the sides of the cell. Friction is eliminated in this way, and the slight leak introduced makes no appreciable difference to the character of the diagrams.

Details of the instrument are shown in the illustrations, where Fig. 1 is a photograph of the whole apparatus, Fig. 2 is a plan, and Fig. 3 a cross-section of the main portion of the instrument. The steel vane *a* is mounted in a cell, *b*, situated on one side of a breech block, *c*. Along its upper edge the vane is attached to a cylindrical bar, *d*, carried in bearings *e*. To the ends of the bar are brazed steel torsion wires *f*, which are secured in clamps *g* adjustable along supports *h*. The vertical end walls of the cell are flat; the lower wall is concave, and the upper wall is slotted to receive the bar *d*. At one end the bar *d* carries a short style *i* of ┐ section and made of an aluminium alloy which enables lightness to be combined with a high degree of mechanical strength.

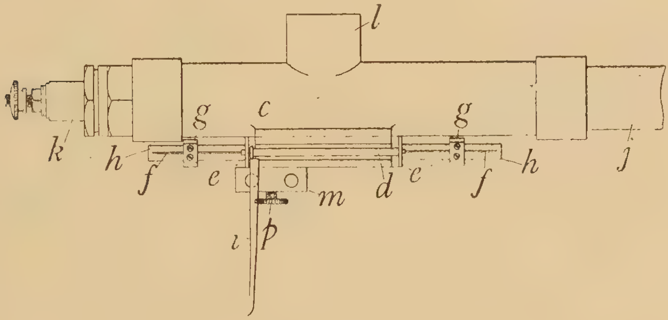


FIG 2.

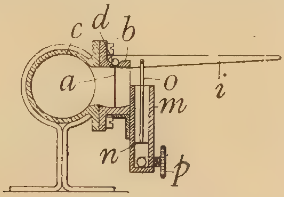


FIG 3.

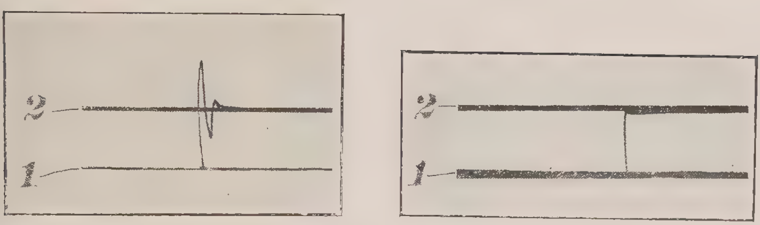
The breech block, which has a bore of 1 in. diameter, is adapted to receive at one end a detachable steel experimental tube, *j*, which may be of any length, and at the other end is fitted with a sparking plug, *k*, when the instrument is used for gas explosions. A branch, *l*, serves for the admission of gas and air mixture, or a platinum coil for the ignition of dust clouds. Both ends of the breech block are made the same, so that an experimental tube can be inserted into each. In the latter case the gas or dust inlet and the ignition device are mounted in another fitting.

The diagram is produced by the style on a smoked paper strip wrapped around a clock-driven drum, and on the same strip is described a time curve by an electrically-driven tuning fork of known frequency.

To make the instrument dead-beat an additional feature is required. For this purpose the idea of incorporating with the cell the pole tips of an electromagnet is attractive, but owing to the mechanical simplicity of the more commonplace dash pot the latter was adopted. In the drawings it will be seen that the dashpot is mounted on the front of the vane cell. It consists of a metal block, *m*, formed with a pair of inter-communicating cylindrical passages in one of which is situated a thin metal disc or piston, *n*, attached to the style by a rod, *o*. A slight clearance is provided around the piston, and the flow of oil between the passages is controlled by a screw, *p*.

The effect of the dash pot is well illustrated at Fig. 4, where *a* is a diagram produced by allowing the vane and style to swing back suddenly from a deflected position indicated by the line 1 to the zero position indicated by the line 2, without the dash pot, and *b* is a diagram produced under similar conditions with the dash pot. Fig. 5 shows at *a* typical explosion diagrams without the dash pot, and at *b* similar diagrams with the dash pot.

Regarding the sensitiveness of the instrument, Fig. 6 gives proof of its capability of responding to slight and rapid pressure variations. If a pneumatic motor horn has its open end covered with a loose plate, leaving at its centre an aperture into which the open end of the experimental tube can fit (the other end of the tube being closed) the low fundamental note of the horn is suppressed and a note of relatively high frequency is obtained on compressing the bulb. The diagram shows the wave-line produced by the instrument in this experiment. It will be observed that the wave-line rises gradually above and descends towards the zero line. This is due to the slight increase of pressure caused by the injected air from the horn bulb.



a *b*

Fig. 4

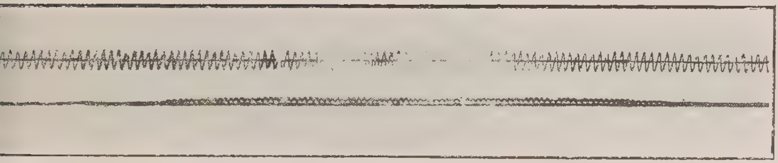


Fig. 6.

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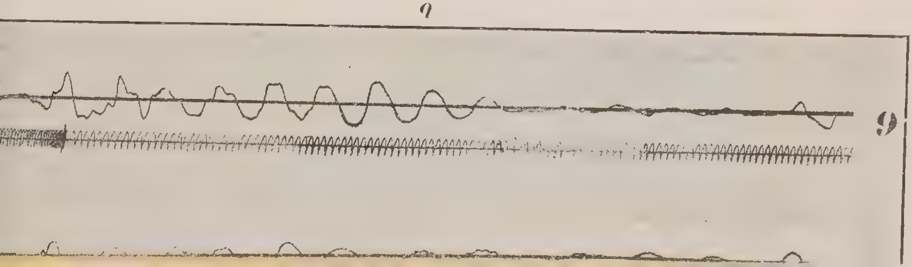
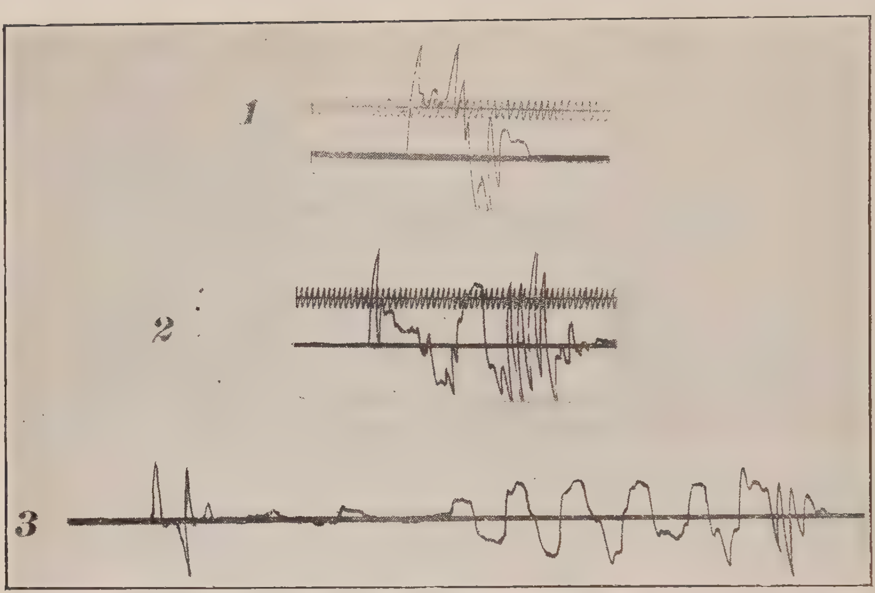
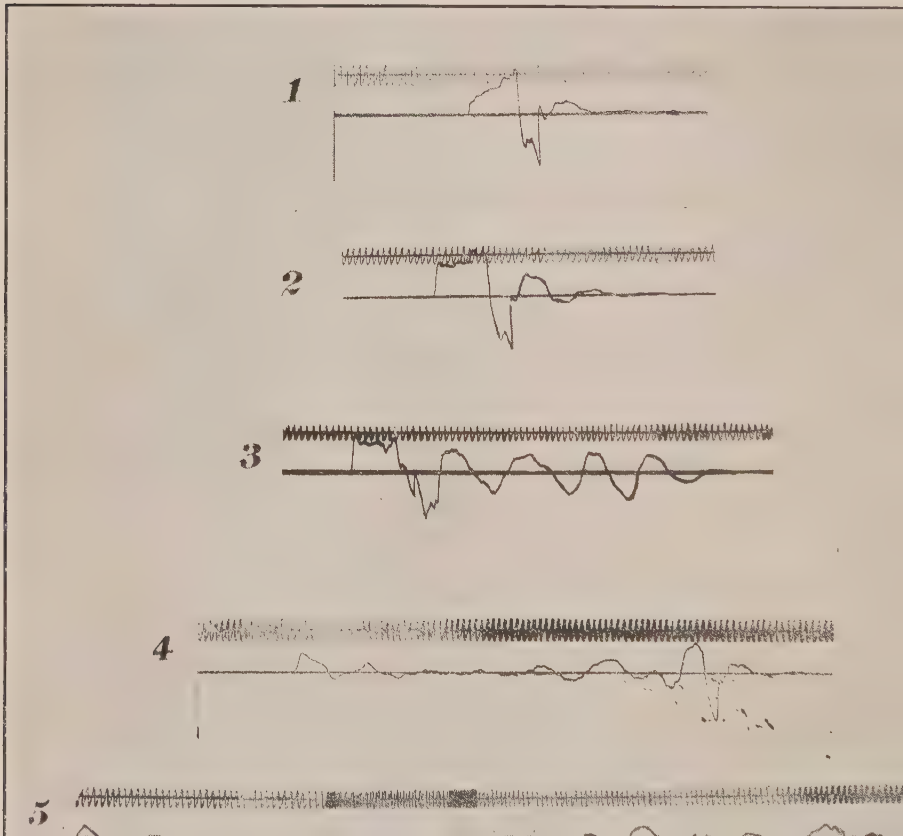
Fig. 5.

b





a



It is not contended that the instrument is capable of giving the precision obtainable by other and more elaborate means, but experience proves it to be a practical device for obtaining by a rapid and inexpensive procedure a large amount of accurate knowledge concerning the phenomena of explosions in tubes. Three defects are inherent in the design, but none of them is serious: In the first place, the ordinates of the curves are not perpendicular to the zero line, but lie along curved lines having the same radius as the style. This introduces a slight distortion, but does not interfere with the accuracy of measurements on the diagrams. In the second place, the damping effect of the dash pot appears to be disproportionately greater during the more rapid parts of the explosion than during the slower parts, and in consequence the ordinates of the diagram corresponding to high pressures rapidly generated are not so great as they ought to be when compared with those of smaller pressures persisting for a longer time. But this defect matters little or nothing when the work of investigation is concerned with the general character of the pressure variation and not the actual pressure obtained at a particular instant. In the third place, the slight leakage past the vane prevents the development of pressures which might be produced if the leakage were absent. This leakage is very slight, and comparison of the diagrams with others taken on a fixed diaphragm instrument show that it has no appreciable effect on the character of the diagrams. Otherwise the instrument appears to give results which are accurate and consistent.

In conclusion, the author desires to express his acknowledgments to E. C. R. Marks, Esq., for facilities afforded in the Marks & Clerk laboratory in Birmingham to develop the instrument, and carry on the work for which the instrument was required.

APPENDIX.

Note on Fig. 5.

The diagrams shown in Fig. 5 are obtained from the explosion of coal gas and air in a 1 in. tube of 10 ft. length. A time line (frequency 250/second) is shown in all the diagrams excepting 3*a*. The diagrams begin at the left-hand end. Positive pressure is indicated above the zero line and negative pressure below that line. No. 1*b* is typical of the diagrams produced when the tube is filled with an explosive mixture. The explosion is attended by a short loud report, and the diagram is always characterised by its shortness, and especially by the sudden transition from positive to negative pressure. This latter characteristic is also found in all diagrams of explosions, whether short or long, when the explosion is accompanied by a sharp report. The same feature is seen in 4, 5 and 6*b*, at the ends of the diagrams. In No. 1*b* the first interval of positive pressure corresponds to the time which elapses before a compression wave is reflected from the closed end of the tube (*i.e.*, the time required for a wave motion to travel four times along the tube). The peculiarity of these short explosions appears to be due to the overtaking of the flame by a compression wave before the flame has travelled any considerable distance through the gas mixture. No. 2*b* is similar to No. 1*b*, excepting that the explosion is followed by a short damped oscillation. In No. 3*b* is shown the kind of curve obtained when the report of the explosion is of a rattling character. Nos. 4, 5 and 6*b* are typical of explosions whose report is of the nature of a rumble terminating in a sharp crack. The crack was more pronounced in 4 and 6 than in 5. The method of producing the diagrams shown in 4, 5 and 6*b* is to allow a quantity of gas to enter the tube, and to let the gas remain for a few seconds before firing. The mixture thus obtained is a poor one, and the diagrams illustrate well the effect of compression by vibration in increasing the explosibility of the mixture. It is not until some pressure has been attained by vibration that the characteristic of a short explosion is obtained, and then the vibration disappears rapidly. Each interval of positive or negative pressure in the more regular parts of the vibrations corresponds to the time required for a wave motion to travel twice the length of the tube. The presence of harmonics is shown clearly in the central portion of No. 5B. As regards Nos. 1 to 3*a*, these illustrate the irregularities introduced by the natural vibration of the vane and style. No. 1*a* corresponds to No. 1*b*, and No. 2*a* to No. 2*b*. No. 3*a* was obtained under similar conditions to Nos. 4 to 6*b*.

ABSTRACT.

The object of this Paper is to describe a mechanical oscillograph for recording the pressure variations which accompany a gas or other explosion in an open tube.

A light steel vane of rectangular form is employed and this is mounted parallel to the explosion tube in a cell presenting a lateral opening to the tube interior. Along three edges the vane is free,

and along the fourth edge it is attached to a torsion wire. The vane is made to fit the cell as closely as possible around its edges without touching the sides of the cell. The diagram is produced by a style on a smoked paper strip wrapped around a clock-driven drum, and on the same strip is described a time curve by an electrically-driven tuning fork of known frequency.

To make the instrument dead-beat a dash-pot is mounted on the front of the vane cell and attached to the style.

DISCUSSION.

The CHAIRMAN asked if the author thought all the phenomena shown in some of his curves were features of the original explosion, or if some of them were due to reflected disturbances from the walls and ends of the tube.

Mr. R. APPLEYARD said the different types of curve shown by the author seemed analogous to the oscillatory and aperiodic types of electric discharge. He asked if the author had tried damping with a magnet and copper plate.

Dr. W. WATSON expressed his interest in the instrument. He mentioned with reference to the statement that a diaphragm cannot be made to give a uniform scale, that with a corrugated diaphragm a quite uniform scale of displacement against pressure could be obtained. He believed the author found it advantageous to have considerable inertia in the moving system, and he thought the inertia would play an even more important role in defining the extent of the vane's motion under the influence of a sudden expansion than the torsional control. He thought that the effect on the resultant curves of varying the inertia ought to be investigated. He supposed that in the curves shown the disturbance was compounded of what he might call the organ-pipe effect, and another effect due to the time taken by the combustion wave to pass along the tube.

The AUTHOR, in reply, said he had considered electromagnetic damping, but rejected it on account of relative cost. There were, however, grave objections to the use of the dash-pot, which undoubtedly modified the shape of the curves. There was no doubt that the resultant disturbances were compounded of a pressure wave along the tube and a surging of the gas as a whole. It was difficult to separate the effects. In most cases—probably in all—the flame seemed to last as long as the needle was vibrating.

XXI. *The Direct Measurement of the Napierian Base.* By ROLLO APPLEYARD, *M.Inst.C.E.*

RECEIVED MARCH 5, 1914.

THE equations of the catenary suggest that it should be possible to derive the Napierian base e from direct measurements of a suspended cord or chain. Such measurements will be found to be a useful exercise for students, especially as an introduction to the employment of hyperbolic functions in solving problems in physics. In Fig. 1, let

a=Length OV of chain, of weight equal to the tension at V, when the catenary is complete on both sides of V.

s=Length of the curved portion VP, where P is any point (x, y) on the catenary.

The familiar equations are

$$y=a\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$s=a\frac{e^{\frac{x}{a}}-e^{-\frac{x}{a}}}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, at the point F on the curve, corresponding to $x=a$, $y=y_1$, we have

$$e=\frac{y_1+s_1}{a}$$

and

$$\frac{1}{e}=\frac{y_1-s_1}{a}.$$

If a is regarded as the unit of length, e is represented in Fig. 1 by FG plus the length of chain FV when straightened out ; for when $x=a=1$, we have

$$e=y_1+s_1$$

and

$$\frac{1}{e}=y_1-s_1.$$

In performing the experiment, the chief difficulty is to ascertain the tension at V without altering the conditions of equilibrium. A pulley running on ball bearings was tried at V with unsuccessful results. Accuracy demands there the equivalent of a frictionless pulley of negligible dimensions, and this requirement is fulfilled by substituting for a pulley a loop of fine thread, as shown along the line VB in Fig. 1. This loop is slipped over the end of the chain up to the point V, and the other end of it is attached to a fixed pin at B. The end D of the chain can then be moved about until VB makes an angle of 45 deg. with the vertical. Then, if V is the lowest point of a catenary, the tangent at V is horizontal, and a is equal to, and at right-angles to, the tension of the catenary at V. Experiment shows however, that the end D of the chain can be moved over a wide range in various directions while retaining the thread VB unaltered in direction along the 45 deg. line, and only the roughest estimate of the value of e

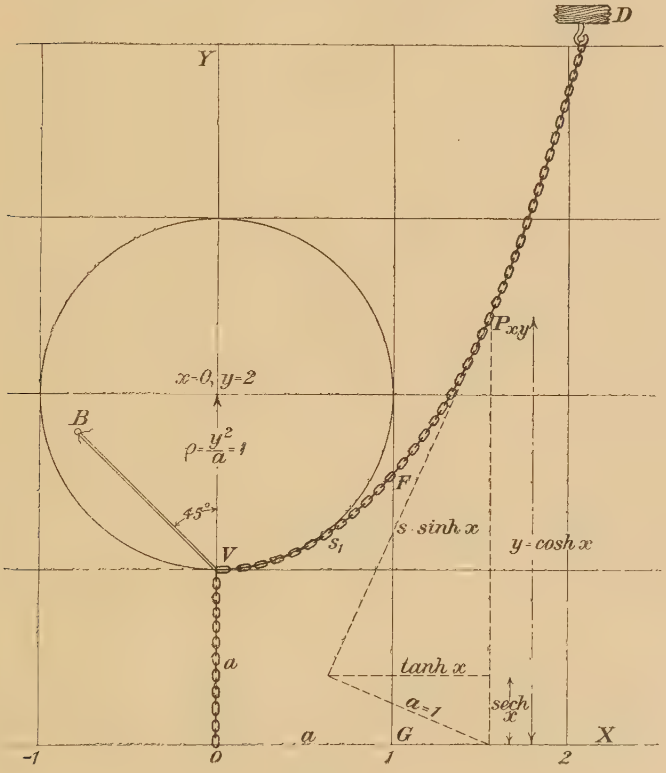


FIG. 1.

can be made with the apparatus in that form. The explanation is that it is impossible to judge by eye when the tangent to the catenary at V is truly horizontal, and a small error in this estimation causes a considerable error in the measurement of y_1. To complete the method, therefore, some additional means must be found for ensuring that the curve reaches its lowest point at V, that its tangent is there horizontal, and that VB remains at 45 deg. to the vertical. This adjustment can be secured by introducing the circle of curvature, the radius of which for any value of y is

$$\rho=\frac{y^2}{a},$$

and the centre of which is always at the point $\left[\left(x-\frac{ys}{a}\right), 2y\right]$.

Hence at V, where $y=1=a$, and where $x=s=0$, the radius is 1, and the centre is at $2y=2$. After drawing the circle, the operation consists in finding the position of the end D, such that while the thread attached to the chain at V is maintained along the line VB, the curved portion of the chain, beginning at V, follows as closely as possible the circle of curvature near V. It, of course, soon departs from the circle, but these two restraints together are sufficient to allow a satisfactory estimate of e to be made. The same apparatus enables sinh x, cosh x, tanh x, and sech x to be found with a piece of chain in like manner, as represented in Fig. 1, and the student is led to see that these functions, which for so long have been forced into association with the hyperbola, are far more easily exemplified by the catenary, and that x then becomes a definite length, instead of an obscure area or an elusive angle.

ILLUSTRATION OF LOGARITHMIC SERIES.

When e has been determined as the sum of the lengths FG and FV, in terms of the length VO as unit, a helpful exercise for the student is to consider what FG and FV separately represent. By direct measurement he finds FG=1·543, which is cosh 1; and FV=1·175, which is sinh 1. Now, if he writes down the series defining e—*i.e.*,

$$e=1+1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}\ \ldots$$

and selects and adds, first the odd terms and then the even terms, he obtains 1·543 and 1·175, as given by the chain measurements; and he recognises here the particular case when $x=1$, of the series and relations

$$\operatorname{Cosh} x=1+\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}+\frac{x^6}{\lfloor 6}+\dots$$

$$\operatorname{Sinh} x=x+\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}+\frac{x^7}{\lfloor 7}+\dots$$

$$e^x=\cosh x+\sinh x=1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}+\dots$$

The point P may then be supposed to move along the chain, and the corresponding changes in the hyperbolic functions of x can be traced in an instructive manner, as represented in Fig. 1.

Logarithms from Chain Measurements.

From the fact that inverse hyperbolic functions can be put into logarithmic form, it appeared likely that chain measurements could be utilised to find logarithms; and this proved to be the case. This can be shown either from equation (1) or from equation (2). Thus, equation (1) can be written

$$e^{\frac{x}{a}}=\frac{1}{a}(y+\sqrt{y^2-a^2}),$$

which gives

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(y+\sqrt{y^2-a^2}). \qquad (3)$$

Similarly, equation (2) gives

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(s+\sqrt{s^2+a^2}). \qquad (4)$$

But Fig. 1 shows that $\sqrt{y^2-a^2}=s$ and $\sqrt{s^2+a^2}=y$. Hence, from (3) or (4),

$$\frac{x}{a}=\log_e\frac{1}{a}+\log_e(s+y), \qquad (5)$$

where $s=\sinh\frac{x}{a}$ and $y=\cosh\frac{x}{a}$.

Now, when $a=1$,

$$x=\log_e(s+y).$$

It follows that at any point along the chain suspended as in

Fig. 1 the Napierian logarithm of any number represented by a length $(s+y)$ is the corresponding x ordinate. For example, at the point $x=2, s=3{\cdot}627$ and $y=3{\cdot}762$; which means that 2 is the Napierian logarithm of $3{\cdot}627+3{\cdot}762=7{\cdot}389$. The extension to the determination of common logarithms involves the modulus, and is less simple. In conclusion, it may be pointed out that the chief object of the experiment is to direct attention to the value of the means afforded by a suspended chain for enabling a number of well-known functions to be presented in a form in which they can be co-related and remembered. A physicist cast upon a desert island and bereft of all but a chain could, to a first degree of approximation, determine the Napierian base, the hyperbolic functions, and the natural logarithms, and he could perform the operations of multiplication and division by mechanical means.

ABSTRACT.

The author described a simple apparatus intended to convey to students an idea of the way in which the base e of the Napierian logarithms enters into physical problems in a specific case of wide application. A small length of chain is allowed to hang from a loop of thread, and the remaining part of the chain is then pulled aside until the thread is at 45 deg. to the vertical. The curved portion becomes a true catenary when the angle between the vertical and curved portions of chain at the attachment of the loop is 90 deg. To ensure that this condition is reached, the circle of curvature of the catenary at that point is drawn, and this is found to have a radius equal to the vertical portion. In these circumstances, if the vertical length is taken as unity, and if its lower end is taken as origin, it is shown that e is the sum of the y-ordinate at $x=1$, and the length of curved chain between the point where that y-ordinate cuts the curve and the top of the vertical portion. The application of this result to a simple representation of the relationship and meaning of hyperbolic functions was also shown, and it was urged that such functions should be studied from consideration of the catenary rather than from the hyperbola.

DISCUSSION.

The CHAIRMAN supposed the object of the Paper was to familiarise students with the properties of the catenary rather than the determination of e, as this was so easily obtained from the formula.

Dr. W. H. ECCLES drew attention to a method of measuring e from the properties of the catenary, which the late Prof. Minchin used to set as a practical problem in the London University examinations.

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